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A

TREATISE

OF THE

THEORY AND PRACTICE

OF

PERSPECTIVE.

WHEREIN

The Principles of that moſt uſeful Art, as laid down by Dr. BROOK TAYLOR are fully and clearly explained, by Means of moveable Schemes, properly adapted for that Purpoſe.

The whole being deſigned as

An Eaſy INTRODUCTION to the Art of DRAWING in PERSPECTIVE,

AND

Illuſtrated by a great Variety of Curious and Inſtructing EXAMPLES.

A Figure in a Picture, which is not drawn according to the Rules of Perſpective, does not repreſent what is intended, but ſomething elſe: ſo that it ſeems to me, that a Picture which is faulty in this Particular, is as blameable, or more ſo, than any Compoſition in Writing which is faulty in Point of Orthography or Grammar.

Dr. BROOK TAYLOR's Preface.

By DANIEL FOURNIER,

Drawing-Maſter, and Teacher of Perſpective.

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BRITISH MUSEUM

TO

Mr. MARQUOIS,

Maſter of the Military Academy at Noreland, near Kenſington Gravel-Pits.

SIR,

THE following Treatiſe of Perſpective being drawn up for the uſe of the noblemen and gentlemen, whoſe military education you ſuperintend ſo ſuccesfully, it is, I preſume, with the ſtricteſt propriety that I inſcribe it with your name.

It adds greatly to your reputation, that you are the firſt man who opened, in a private way, an academy of this kind. How very important ſuch inſtitutions are judged by the government, appears from the Royal Academies of Woolwich and Portſmouth, founded under the auſpices of his late Majeſty King GEORGE II.

The valour of the Engliſh has been ever acknowledged, as the French have often experienced to their coſt: but it was long obſerved, that

the military ſkill of the French officers was far ſuperior to that of the Engliſh, and this through want of military academies, where our noble and generous youth might be taught thoſe arts, which only can enable them to ſhine both in the field and on the ocean.

How uſeful a member, Sir, you are to ſociety, appears from the excellent pupils you are daily forming: 'tis to your friendſhip that I owe the great honour I have to inſtruct them in the art of drawing and perſpective. For this, and many other favours from you, gratitude obliges me to wiſh all imaginable ſucceſs to your laudable endeavours; and to aſſure you, that no one can be with more perfect eſteem,

SIR,

Your moſt humble,

and moſt obedient ſervant,

DANIEL FOURNIER.

THE PREFACE.

AS Dr. Brook Taylor's Perſpective is undoubtedly the moſt excellent performance of its kind hitherto made public, I choſe rather to attempt an explanation of his principles, than to offer any thing of my own, with regard to the theory of this moſt uſeful art; and chiefly becauſe the generality of thoſe who apply themſelves to the ſtudy of that treatiſe, without being firſt acquainted with geometry, find great difficulty in conceiving the author's deſign, as all his ſchemes are drawn in plano: I have, to obviate this difficulty, introduced moveable ſchemes in the following work, ſo contrived, as to be raiſed up and placed according as the nature of the problem may require.

In the practical part, which begins at page 32, I have defined and made uſe of the terms, ground line, horizontal line, &c. This I did for the conveniency of thoſe who may have already learned the theory of perſpective, from ſuch authors as have treated thereon agreeable to theſe definitions, and becauſe painters commonly uſe the horizontal line as the boundary of the picture.

As the art of perſpective conſiſts in the knowledge of truly repreſenting an object according to its natural appearance, ſo whatever is not drawn agreeable the rules thereof, cannot poſſibly repreſent what is intended, but ſomething elſe: it is true, that as the art of painting conſiſts of two parts, the inventive and the executive, the former may indeed be varied according to the taſte

and

and genius of the artiſt, in the diſpoſition of the parts of his ſubject; but the latter is wholly confined and ſtrictly tied to the rules of art, and therefore in this the artiſt is not to take any liberties whatſoever; for what is perfectly juſt in the real original objects themſelves, can never appear defective in a picture, where thoſe objects are repreſented according to the true rules of perſpective.

Notwithſtanding the principles upon which Dr. Taylor has founded his treatiſe ſeem ſo very obvious as not to admit of any objection; yet ſome modern writers upon this ſubject have ſo far deviated from his theory, as to inform us, that a circle, when viewed in an oblique poſition, cannot poſſibly take the appearance of a true mathematical ellipſis. I ſhall not at preſent enter into a confutation of the reaſons they give to ſupport the truth of this extraordinary aſſertion, but only point out wherein (I think) they are miſtaken. That the oblique ſection of a cone may be an ellipſis, is certainly true: I ſay may be an ellipſis, becauſe in ſome caſes it may be an hyperbola; wherefore, if the cutting plane be ſuppoſed to be the plane of the picture, the repreſentation of a circle thereon muſt be either a circle, ellipſis, or a right line, and that according to the different ſituations of the original circle with regard to the plane of the picture; if after having found the elliptical repreſentation of an original circle, we find that of its center, it will not coincide with the center of the ellipſis; and this I apprehend may have deceived thoſe who have imagined the oval repreſentation of an oblique circle would not be of equal curviture in oppoſite parts of its circumference.

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OF

LINEAR PERSPECTIVE.

IN order that the learner may form a clear idea of the inveſtigations relating to the theory of *Perſpective* contained in the enſuing pages, it may be neceſſary to premiſe the following geometrical Definitions and Problems.

DEFINITIONS.

1. *Magnitude* is whatever is extended, and therefore may be conceived to be contained or limited by ſome certain term or terms.

2. A *point* is that which has no parts, conſequently of itſelf no magnitude, and may be conſidered as indiviſible.

3. A *right line* is the neareſt diſtance between two points.

4. If two right lines inclined to each other be produced till they meet, they will form a plane or *right lined angle*. Thus the *angle* formed by the lines BD, CE, Plate I. Fig. 1. produced, is denoted by BAC.

N. B. An angle being generally denoted by three letters, the middle one always repreſents the angular point.

5. When a ſtrait line ſtands on another, ſo that it does not incline to one end more than the other, but makes the angles on both ſides equal; then are theſe called *right angles*: and the lines are ſaid to be *perpendicular* to each other; as BD Plate I. Fig. 2. is perpendicular to FE.

6. Every angle greater than a right one, is called an *obtuſe angle*; and every angle leſs than a right angle, is called an *acute angle*.

7. A plane ſurface terminated by three right lines, is called a *plane triangle*.

8. When the three ſides of a triangle are equal, it is called an *equilateral triangle*.

9. When only two ſides are equal, an *iſoſceles triangle*.

10. When the three ſides are unequal, a *ſcalene triangle*.

11. When one angle of a triangle is a right angle, it is called a *right angled triangle*.

12. Any one ſide of a triangle may be called the *baſe*, and the angular point oppoſite to the baſe is called the *vertex*.

13. In a right-angled triangle, the ſide oppoſite to the right angle is called the *hypothenuſe*.

14. A plane contained by four right lines, is called in general a *quadrilateral* figure; and when the four ſides are equal as well as the four angles, it is called a *ſquare*.

15. If the four angles are equal, and only the oppoſite ſides, a *rectangle*.

16. If the oppoſite angles are only equal, and the oppoſite ſides, a *parallellogram*.

17. A plane figure of above four ſides is called, in general, a *polygon*.

18. A *circle* is a plane figure bounded by one continued line called the *circumference*, which is every where equally diſtant from a point within the circle, called the *center*.

19. Any line drawn from the center to the circumference is called a *radius*; the double of which is the *diameter* of the circle.

20. The meaſure of an angle is the arc deſcribed from the angular point as center with any radius, and terminated by the lines which form the angle.

21. A line which touches a circle in one point only, is called a *tangent*; and the point where it touches, the point of *contact*.

22. A polygon is called *regular*, when all its ſides are equal as well as all its angles; and *irregular*, when they are not equal.

23. A polygon is ſaid to be *inſcribed* in a circle, when all its angles touch the circumference; and circumſcribed about a circle, when all its ſides touch the circle.

Polygons are diſtinguiſhed by the number of their ſides, viz.

5 Pentagon,	9 Enneagon,
6 Exagon,	10 Decagon,
7 Eptagon,	11 Ondecagon,
8 Octagon,	12 Dodecagon,

PROBLEM 1.

To divide a given right line into two equal parts.

Let the line given be AB, Plate I. Fig. 3. Set one foot of the *compaſſes* in A, and with any convenient diſtance ſweep two *arches* one above and the other below the given *line*; then ſet one foot of the *compaſſes* in B, and with the ſame diſtance as before croſs the aforeſaid *arches*, join the points of interſection with a right line, and it will cut AB in two equal parts.

PROBLEM 2.

To raiſe a perpendicular from any point propoſed, in a given right line.

Let the given line be AB, Plate I. Fig. 4, and the *perpendicular* to be raiſed from the point C; to do which, ſet off two equal diſtances from the point C, as CR, CS: then the *compaſſes* being opened to any convenient diſtance greater than CR or CS, with one foot of the *compaſſes* in the point R deſcribe the *arch* DE: then with the ſame extent, and one foot of the *compaſſes* placed in the point S, deſcribe the *arch* FG; then

draw from C a right line through the interſection of the two *arches* FG, DE, and it will be the *perpendicular* required.

PROBLEM 3.

To let fall a perpendicular from a point aſſigned, to a given right line.

Let AB, Plate I. Fig. 5. be a right line given, and C the point from whence the *perpendicular* muſt fall.

Set one foot of the *compaſſes* in C, and opening them to a proper diſtance deſcribe the arch RS; then by Prob. 1, divide RS into two equal parts in D, join the points C, D, and CD will be the *perpendicular* required.

PROBLEM 4.

To draw a right line parallel to a given right line, and alſo through a point given.

Let AB, Plate I. Fig. 6, be the given line, and C the point given. From C let fall, by the laſt Problem, the perpendicular CD upon the line AB. Take any point, as E in AB, in which ſet one foot of the *compaſſes*, and with an opening equal to CD, deſcribe an *arch* FG; lay a ruler, ſo as juſt to touch this *arch* and the point C, then draw the line CH, it will be the *parallel* required.

PROBLEM 5.

To divide a given right line into any number of equal parts.

Let AB, ~~Fig.~~ I. ~~Plate~~ 7, be the given line.

From the point A draw a right line AC at pleaſure; from the point B draw a right line BD parallel to AC, ſet off along AC and BD from the points A and B, as many equal parts AE, EF, FG, &c. and BK, KL, LM, MN, &c. towards C and D reſpectively, as are one leſs than the number into which you intend the given line to be divided; join the laſt point in AC with the firſt in BD, and alſo join the other points I, L, G, N, F, O, &c. and the right lines KK, IL, HM, GN, &c. will divide AB into the deſired number of equal parts at the points k, i, h, g, &c.

PROBLEM 6.

To divide a given angle ACB, Plate I. Fig. 8, *into two equal parts.*

From the angular point C as center, deſcribe an arch with any radius, meeting the legs of the angle at A, B: from theſe points, as centers, deſcribe two *arches* with the ſame radius, ſo as to interſect each other at D; then the line drawn through this interſection and the angular point C, will biſect the given angle.

PROBLEM 7.

To draw a line CA, *ſo as to make an angle* ACB, *with a given line* CB, *equal to a given angle* a c b, Plate I. Fig. 9.

From the points C, c, as centers, deſcribe two arches with any radius at pleaſure: make the arch AB equal to the arch a b; and the line drawn through the points C, A, will form the angle required.

Note, The circumference of every circle, whether great or ſmall, is ſuppoſed divided into 360 equal parts, called degrees, each degree into 60 equal parts, called minutes. Hence a quadrant, or the meaſure of a right angle, is 90 degrees, being the fourth part of the circumference; a ſemicircle, or the meaſure of two right angles, is 180 degrees, being the one half of the circumference.

PROBLEM 8.

To deſcribe the circumference of a circle, through three given points A, B, C, Plate I. Fig. 10.

Join theſe points by two lines AB, BC, biſect them with the perpendiculars DO, EO; their point of interſection O, will be the center of the circle deſired.

PROBLEM

PROBLEM 9.

To inſcribe a ſquare in a circle. Plate I. Fig. 11.

Draw the diameter AB, to which draw the diameter CD perpendicular; then the lines joining the extremities of theſe diameters will be the ſquare ABCD required.

PROBLEM 10.

To inſcribe a regular polygon of a given number of ſides in a circle.

Draw two *radii* from the center of the propoſed circle, making an angle equal to the *angle at the center of a polygon* anſwering to the aſſigned number of ſides (as in the table annexed): join the two points in the circumference, where the *radii* interſect it, add that line or *chord* will be the ſide of the required polygon.

A TABLE of regular POLYGONS.

Polygon.	Angle at the Center.	Angle made by the Sides.
5	72	108
6	60	120
7	$51\frac{3}{7}$	$128\frac{4}{7}$
8	45	135
9	40	140
10	36	144
11	$32\frac{8}{11}$	$147\frac{3}{11}$
12	30	150

They

The angle at the center of a regular polygon is found by dividing 360 degrees by the number of the ſides: thus 360 divided by 5 gives 72 degrees for the angle at the center of a Pentagon; and 360 divided by 6 gives 60 degrees for the angle at the center of an Exagon.

But the angle made by two adjacent ſides of a polygon is found by ſubtracting the angle of the center from 180 degrees: thus from 180 degrees take 72, there remains 108 degrees, which is the angle made by the ſides of a Pentagon; and if from 180 we take 60, (the angle at the center of an Exagon) there will remain 120, the angle made by the ſides of an Exagon: in the ſame manner may the remaining numbers in the third column of the foregoing table, be found.

Having furniſhed the reader with the moſt neceſſary geometrical Problems, for the underſtanding of Perſpective, we ſhall next proceed to the Definitions thereof.

DEFINITION I.

In order to have a clear idea of the principles of this art, we are to conſider that a picture drawn perfectly true, and placed in a proper poſition, ought ſo to appear to the ſpectator, that he ſhould not be able to diſtinguiſh the repreſentation from the real original objects actually placed where they are repreſented to be. To produce this effect, it is neceſſary that the rays of light ought to come from the ſeveral parts of the picture to the ſpectator's eye with the ſame circumſtances of direction, ſtrength of light and ſhadow, and colour, as they would do from the correſponding parts of the real objects ſeen in their proper places. Thus, Plate II. Fig. 1. let O be the ſpectator's eye, FH the plane of the picture, which, for more eaſy conception, we may ſuppoſe to be tranſparent, ABCDE an original cube; then if we imagine lines to be drawn from O to every part (in view) of the ſolid ABCDE, their interſection with the plane FH will mark thereon the repreſentation a b c d e of the original figure ABCDE.

2. *Cone of rays.* Lines drawn from the ſeveral parts of any figure, and continued ſo as to paſs through one and the ſame point, conſtitute a cone of rays; and when that point is conſidered as the eye of a ſpectator, it is called the *optic cone.*

3. *Ichnography.* When a ſyſtem of rays (parallel to each other and perpendicular to the horizon) coming from the ſeveral points in any figure, is cut by a plane parallel to the horizon, the projection thereon is called the *ichnography of the ſaid figure.*

4. *Orthography.* When a ſyſtem of rays (parallel to each other and to the horizon) coming from the ſeveral points in any figure, is cut by a plane perpendicular to the horizon, the projection thereon is called the *orthography of the ſaid figure.*

Theſe are the *common* definitions of the terms *ichnography* and *orthography*; but in the following pages we ſhall uſe them to ſignify any two projections that are made by ſyſtems of parallel rays, when thoſe ſyſtems are perpendicular to each other, and to the planes on which the projections are made, without having any regard to their ſituation with reſpect to the horizon.

5. *Schenography.* When the optic cone is cut by a plane, the repreſentation of the propoſed figure thereon is called the *ſchenography* thereof. It is evident that the ſhadows of figures are this projection, when the light is conſidered as a ſingle point: though in the caſe of the ſun or moon, that point being at an infinite diſtance (as to all ſenſe), the projecting rays are parallel to each other.

6. *Point of ſight* is the vertex of the optic cone, or it is that point where the ſpectator's eye ſhould be placed to look upon the picture.

7. *Diſtance of the picture.* The point in which a perpendicular let fall from the *point of ſight* upon the picture cuts it, is called the *center* of the picture; the diſtance between the *center* and *point of ſight* is called the *diſtance of the picture.*

8. *Directing plane* is that plane which paſſes through the *point of ſight* parallel to the picture.

9. *Original object*; the real object (whether it be point, line, ſurface, or ſolid,) placed in the ſituation it is repreſented to have by the picture.

10. *Original plane.* The plane wherein is ſituated any original point, line, or plane figure, is called the *original plane.*

11. *Interſection of an original line* is that point wherein the ſaid line (continued if need be) cuts the picture.

12. *Interſection of an original plane* is that line wherein the ſaid plane cuts the picture.

13. *Directing point of an original line* is that point wherein the ſaid line cuts the *directing plane.* And a right line joining the directing point and point of ſight, is called the *director* of that original line.

14. *Directing line of an original plane* is that line wherein an original plane cuts the directing plane.

15. *Parallel of an original line* is a right line drawn through the point of ſight parallel to the ſaid original line.

16. *Parallel of an original plane* is a plane paſſing through the point of ſight parallel to the ſaid original plane.

17. *Vaniſhing point* is the point where the parallel of any original line meets the picture, and is called the *vaniſhing point* of that line; the diſtance between the vaniſhing point and the point of ſight is called the *diſtance* of that vaniſhing point.

18. *Vaniſhing line* is that line wherein the parallel of any original plane interſects the picture. From the point of ſight let fall a perpendicular to the vaniſhing line; the point where that vaniſhing line is ſo cut is called the *center* of it, and the diſtance between the center and point of ſight is called the *diſtance* of that vaniſhing line.

19. *Repreſentation of any figure* is the projection of that figure. In order to comprehend the ſenſe of theſe definitions more fully, let the planes ABIC, DEO, Plate III. Fig. 1. be ſo raiſed up as to ſtand parallel to each other, and bring the plane ACO parallel to the plane FGH: this done, imagine the plane ABIC to be the ſurface of the picture, O to be the point of ſight, (Def. 6. Perſpective) the plane DEO to be the directing plane, (Def. 8.) FG to be an original line (Def. 9.) in the original plane FGH, (Def. 10.) cutting the picture in BI and the directing plane in DE: and let ACO cut the picture in the line AC, and ſuppoſe OV to be drawn parallel to the original line FG, and cut the picture in V: and let FG (produced) cut the picture in B, and the directing plane in D; then is B the interſection of the original line FG, (Def. 11.) D its directing point, (Def. 13.) and V its vaniſhing point, (Def. 17.) and OD its director, (Def. 13.) and OV is the diſtance of the vaniſhing point V (Def. 17.); BI is the interſection, (Def. 12.) DE the directing line, (Def. 14.) ACO the parallel, (Def. 16.) and AC the vaniſhing line (Def. 18.) of the original plane FGH. From O let fall a line perpendicular upon the vaniſhing line AC, and the point wherein the ſaid perpendicular meets AC, as S, will be the *center* of the picture, (Def. 18.) and SO will be the diſtance of the vaniſhing line AC (Def. 18.).

AXIOM 1.

The common interſection of the two planes is a ſtraight line.

AXIOM 2.

If two ſtraight lines meet in a point, or are parallel to one another, there may be a plane paſſing through them both.

AXIOM 3.

If two parallel ſtraight lines are cut by a third line, they will all three be in the ſame plane; that is, a plane paſſing through any two of them will alſo paſs through a third.

AXIOM

Axiom 4.

Every point in any ſtraight line is in any plane that line is in.

Theorem 1.

A line drawn from the center of the picture to the center of a va niſhing line, is perpendicular to that vaniſhing line.

Demonstration.

Raiſe up the plane AC (Plate III. Fig. 2.) perpendicular to the plane of the picture AFG; draw OC perpendicular to AF, and move the triangle OSC about OC as an axis, until its baſe CS is perpendicular to a given vaniſhing line, as AB; then will OS be perpendicular to ASB, and conſequently (Def. 18.) S is the center of the vaniſhing line propoſed.

Corollary.

The diſtance OS of any vaniſhing line ASB, is the hypothenuſe of a right-angled triangle, whoſe legs are the diſtance of the picture OC, and the diſtance CS between the center of that vaniſhing line and the center of the picture.

Theorem 2.

The perſpective repreſentation, or projection of any object, is the ſame as the ichnographic projection of it on the plane of the picture, the point of ſight being the vertex of the optic cone.

Demonstration.

For, by the explanation of the principles in Definition 1. the light muſt come to the ſpectator's eye O, (Plate II. Fig. 1.) in the ſame direction from any point *a* of the projection as it would do from the correſponding point A of the original object; therefore it is evident that the rays *a*O and AO are in one and the ſame ſtraight line. Whence it follows that the projection *a* is the interſection of the picture with the ray AO, and the

whole projection *abcde* is the ſchenography of the original figure ABCDE, made by the optic cone OABCDE, whoſe vertex is the point of ſight O.

Corollary 1.

The projection of a ſtraight line is a ſtraight line. For conceive a plane paſſing through the point of ſight, and the original line to interſect the picture, it is then evident that the interſection can only be a ſtraight line. As *de* is the interſection of the picture with the triangle ODE, and conſequently a ſtraight line (by Ax. 1.).

Corollary 2.

The original of a projection may be any object that will produce the ſame cone of rays. Thus the original of the projection *de* may be any line *de*, which produces the optic cone ODE, as well as the line DE.

This being ſo, it may reaſonably be aſked, whence it happens that figures drawn on a picture appear to be what they are deſigned to repreſent. The reaſon is, becauſe the mind has acquired a habit of judging objects, that are ſo and ſo related, have ſuch and ſuch colours, and are ſo and ſo enlightened and ſhaded, to be of ſuch a ſhape and alike ſituated. Theſe circumſtances are all of them neceſſary to make a picture complete, though the ſimple drawing is ſometimes almoſt ſufficient, upon account of the relation of the parts; as in a pavement, where all the ſtones appear to be ſquare, though they are repreſented by very irregular figures. I ſay, it is the relation of the parts which produces this effect; for the repreſentation of any one of the ſingle ſquares would hardly appear to be ſquare, were there no other objects to biaſs the judgment by their relation to it.

Theorem 3.

The projection of a ſtraight line not parallel to the picture, paſſes thro' both its interſection and vaniſhing point.

Demon-

DEMONSTRATION.

Imagine an indefinite plane (paſſing through the point of ſight and the original line) to cut the picture, then it is very obvious that in the line which forms their interſection, that of the original line with the picture will always be found: but by Def. 17. the line determining the vaniſhing point will alſo be in this plane, therefore the projection of an original line FG (Plate III. Fig. I.) paſſes through its interſection B and vaniſhing point V.

This Theorem being the principal foundation of all the practice of *perſpective*, the learner would do well to make it very familiar to him. In order to which I have again repreſented the meaning of it in Plate II. Fig. 1. where the projection *bc* meets the original line BC in its interſection K, and paſſes alſo through its vaniſhing point V, which is produced by its parallel OV.

N. B. When the original line itſelf paſſes through its vaniſhing point, the whole projection of it will be that point; ſo that in that caſe the line may be ſaid to vaniſh: this is one reaſon for uſing that term. Another reaſon is, that the further any object is off upon any line, the ſmaller is its projection, and, at the ſame time, the nearer to this point, and when it comes into this point, its magnitude will entirely vaniſh, becauſe the original object is at an infinite diſtance. This is eaſily conceived, by imagining a perſon going from you in a long walk, who appears to be ſmaller and ſmaller the further he goes. The reaſon of this diminution will appear from the following *Corollaries*.

COROLLARY I.

Projections of original lines that are parallel to each other, but not to the picture, paſs through the ſame vaniſhing point: for they have but one parallel common to them all, and conſequently but one vaniſhing point.

As

As in Plate II. Fig. I. where theprojection *da* and *cb* of the parallel lines DA and CB meet in their common vaniſhing point V.

COROLLARY 2.

The center of the picture is the vaniſhing point of lines perpendicular to the picture.

THEOREM 4.

The projection of an original line parallel to the picture is alſo parallel to that original line.

DEMONSTRATION.

Imagine a triangular plane, whoſe vertex is at the ſpectator's eye, and its baſe the propoſed original line, to be cut by the picture. It is evident the repreſentation upon the picture will be parallel to the baſe of this triangular plane, becauſe the picture is ſuppoſed to be ſo; therefore if EF, Plate III. Fig. 3. be the picture, AB an original line parallel to it, then will its projection *ab* be parallel to the original AB, O being the point of ſight.

COROLLARY 1.

Lines parallel to one another, and to the picture, are projected into lines parallel among themſelves. Thus *ab* and *dc* are parallel to each other, and to their originals AB and CD.

COROLLARY 2.

The projection *abcd* of any plane figure ABCD parallel to the picture, is ſimilar to its original. For draw AC and *ac*, then will the triangles ACB, *acb*, be ſimilar, as will alſo the triangles ADC, *adc*, conſequently ABCD and *abcd* are ſimilar.

COROLLARY 3.

AB : *ab* : : diſtance of the picture : diſtance between the point of ſight and the plane of the original figure; that is, (O*g*G being drawn perpendicular to thoſe two planes) AB : *ab* : : O*g* : OG.

THEOREM

THEOREM 5.

The projection of a line is parallel to its director. For the lines OF, OG, OD, *fg*. (Plate III. Fig. 1.) are all in the ſame plane; but the directing plane ODE is parallel to the plane of the picture ABIC (Def. 8.); therefore the director OD is parallel to the projection *fg*. Eu. 11. 16.

COROLLARY 1.

The projections of lines that have the ſame director are parallel to each other.

COROLLARY 2.

If the original line is parallel to the picture, its director is ſo too, and conſequently is in the parallel of any plane paſſing through that original line; and therefore the vaniſhing line of that plane, and the projection of the line are parallel to one another.

THEOREM 6.

The vaniſhing line, interſection, and directing line of any original plane, are parallel to each other.

For the planes OVC, DFH, (ſee the laſt Figure) are parallel, (Def. 16.) as alſo ODE and CAB (Def. 8.); therefore the vaniſhing line CV, interſection IB, directing line ED, are parallel to each other.

COROLLARY.

The triangles O*f*V, DOF, are ſimilar, and DO= BV, therefore *f*V : VO : : BV : DF.

THEOREM 7.

All lines in any original plane have their vaniſhing points in the vaniſhing line of that plane.

DEMONSTRATION.

For as all the original lines are in the ſame plane, the lines which determine their vaniſhing points, will interſect the picture in the vaniſhing line of the original plane.

COROLLARY.

COROLLARY 1.

Original planes parallel to each other have the ſame vaniſhing line.

COROLLARY 2.

The vaniſhing point of the common interſection of two original planes, is the common interſection of their vaniſhing lines.

COROLLARY 3.

The vaniſhing line of a plane perpendicular to the picture, paſſes through the center of the picture.

THEOREM 8.

Interſections of all lines in the ſame original plane, and alſo the line which determines the interſection of that original plane with the picture, are in that plane.

This needs no DEMONSTRATION.

COROLLARY 1.

The interſection of two original planes, which is not parallel to the picture, being produced, will interſect the picture in that point which is the common interſection of the two planes therewith.

COROLLARY 2.

If the common interſection of ſeveral planes is parallel to the picture, then thoſe planes have parallel interſection therewith, and have alſo parallel vaniſhing lines.

PROBLEM 1.

Having given the center and diſtance of the picture, to find the projection of a point, whoſe ſeat * on the picture, with its diſtance from it, are given.

Let O be the ſpectator's eye, Plate III. Fig 2. S the center of the picture, and *b* the given ſeat of the original point A; then if we imagine a

* By *ſeat on the picture*, we mean that point wherein a perpendicular falling on the picture from the point to be projected, interſects it.

line drawn from O to A, it will cut the plane of the picture ſomewhere in that line which joins the center of the picture and ſeat of the original line, as at *a*, which is therefore the repreſentation of the point A, by Theo. 2. The triangles A*ab* O*a*S are ſimilar; whence S*a* : *ab* :: SO : A*b*.

Corollary 1.

The point A may be found by a ſcale and compaſſes, dividing the line S*b* in *a*, ſo that S*a* may be to *ab*, as the diſtance of the picture OS, is to the diſtance A*b* of the original point from its ſeat *b*.

Corollary 2.

By this Problem the projection of any right line may be found; for it is no more than finding the projection of any two points therein, and drawing a right line through thoſe two projections.

Problem II.

To find the projection of a line, its vaniſhing point and diſtance; having given its ſeat, interſection, and the angle it makes with its ſeat, and the center and diſtance of the picture.

Let W (Plate IV. Fig. 1.) be a point in the given line, E its ſeat on the picture, and D its interſection.

Make SO equal to the diſtance of the picture, draw SV parallel to DE and perpendicular to SO, join the center of the picture and ſeat of the point W by drawing ES; then it is evident that the projection of W will be in this line: but to determine the exact place, we muſt divide ES in the proportion of WE to SO, which may be done thus: MakeET=EW, SM=SO; draw TM, and the point of interſection *w* will be the place required: join the points D, *w*, and D*w* will be the indefinite repreſentation of the line propoſed. If from the point of ſight, a right line be drawn parallel to DW, it will interſect the picture in the vaniſhing point thereof, and its diſtance will be equal to OV.

Corollary 2.

If the vaniſhing point and interſection of the original line DW be joined by a right line, it will cut ES in *w*, the projection of the original point W.

Problem III.

Having given the projection of a line, and its vaniſhing point, to find the projection of the point that divides the original line in any given proportion.

Let AB (Plate IV. Fig. 2.) be the given projection of the line to be divided, and V its vaniſhing point. Draw at pleaſure VO and *ba* parallel to it; and through any point V of the line VO, draw OA and OB cutting *ba* in *a* and *b*. Divide *ab* in *c* in the proportion given, and draw O*c* cutting AB in C. Then ſhall C be the projection ſought, the original of BC being to the original of CA, as *bc* is to *ca*.

For OV being parallel to *ba*, *ba* may be conſidered as the original line, and OV as its parallel, and conſequently O as the point of ſight, and *a*O, *b*O, *c*O, as viſual rays projecting the points A, B, C.

Corollary.

CA × BV : CB × AV : : *ca* : *cb*; for draw *ef* through C parallel to *ab*, then we ſhall have

$$\left.\begin{array}{l} AC : AV :: eC : OV \\ BV : CB :: OV : Cf \end{array}\right\} \text{whence } AC \times BV : AV \times CB :: eC : Cf :: ca : cb.$$

Therefore the point C may be found by a ſcale and compaſſes, making CA : CB : : AV × *ca* : BV × *cb*.

Problem IV.

Having the projection of a line, and its vaniſhing point, from a given point in that projection, to cut off a ſegment, that ſhall be the projection of a given part of the original of the projection given.

Let

Let AB (Plate IV. Fig. 3.) be the projection given, V its vaniſhing point, and C the point from whence is to be cut off the ſegment. Draw at pleaſure VO, and *abc* parallel to it, and from any point O in VO, draw OA, OB, OC, cutting *a b* in *a*, *b*, *c*. Make *cd* to *ab*, as the given part is to the original of AB, and draw O*d* cutting AB in D: then will CD be the ſegment ſought.

The Demonſtration of this problem is obvious from that of the foregoing one.

COROLLARY

The point D may be found by a ſcale and compaſſes, making DC : DV : : $dc \times AB \times CV : ab \times AV \times BV$. For, per laſt Prob.

$$\left.\begin{array}{l} AB \times BV : AV \times BC :: ab : bc. \\ BC \times DV : BV \times CD :: bc : dc. \end{array}\right\} \text{whence } AB \times CV \times DV \times dc = AV \times BV \times CD \times ab.$$

and therefore DC : DV : : $dc \times AB \times CV : ab \times AV \times BV$.

N. B. The foregoing Problem may be conſidered as a particular caſe of this, *viz.* when the point C in this coincides with one of the points A or B.

PROBLEM V.

Having given the center and diſtance of the picture, to find the vaniſhing line (with its center and diſtance) of a plane, whoſe interſection is given, with the angle of its inclination to the picture.

Let C (Plate III. Fig. 2.) be the center of the picture. Draw CG parallel to the interſection of the original plane with the picture, and equal to the diſtance of the picture, and from C erect the indefinite right line CS perpendicular to CG; draw GS, making an angle SGC equal to the complement of the angle of inclination of the original plane to that of the picture; draw through S the right line ASB. Then will ASB be the vaniſhing line ſought, S its center, and GS its diſtance.

Becauſe GC=CO, and the angle GSC equal to the given inclination of

the original plane and picture (by conſtruction), it is evident that the right angled triangle GCS is equal to the moveable right-angled triangle COS; therefore, by Defin. 18. ASB is the vaniſhing line, and S its center.

N. B. Taking GC for radius, CS is the cotangent, and GS the coſecant of the inclination of the original plane to the picture.

PROBLEM VI.

Having given the interſection of an original plane, with its vaniſhing line, its center and diſtance; to find the projection of any line in the original plane, having the original figures drawn out in their juſt proportions.

Let DF (Plate IV. Fig. 4.) be the interſection given, HG the vaniſhing line, and S its center. Draw SO perpendicular to GH, and equal to the diſtance of the vaniſhing line GH, and let the ſpace X be the original plane, ſeen on the reverſe, as objects appear in a looking-glaſs; the ſpace Y being the parallel plane in the ſame manner folded down on the picture: and let AB be the original line whoſe projection is ſought. Let AB cut the interſection in D, and draw OG parallel to AB, cutting the vaniſhing line in G. Draw DG, which will be the indefinite projection of AB. Through A and B draw at pleaſure AC and BC meeting in C, and in the ſame manner find their indefinite projections FI, and EH, cutting DG in *a* and *b*. Then will *ab* be the determinate projection of AB, *a* being the projection of the extremity A, and *b* the projection of the extremity B.

OTHERWISE,

Suppoſe KL to be the original line given. Having found its indefinite projection QG, as before, draw OK and OL, cutting it in *k* and *l*, which will be the projections of the extremities K and L.

Otherwiſe, by the DIRECTORS,

Let DF (Plate IV. Fig. 5.) be the interſection given. And let the original plane be folded down on the picture, ſo as to bring the directing line into the plane HI, the diſtance between AF and HI being equal to the

diſtance

diſtance of the vaniſhing line given. Let alſo O be the point of ſight brought into the picture at the ſame time along with the directing plane HIO. To find the indefinite projection of any original line AB, continue it till it cuts EF in F, and HI in G; then draw OG, and F*a* drawn parallel to it will be the indefinite projection ſought. Then finding in the ſame manner the indefinite projection E*d* of any other line, AD paſſing through A, by its interſection with F*a* is got the projection *a* of the extremity A. And in the ſame manner is got the other extremity *b*; or thoſe extremities might be found by drawing lines from A, and from B to O, as in the foregoing conſtruction.

To prove the truth of theſe operations, imagine the figures (Plate IV. Fig. 4.) to be folded in DF and HG, and (Plate IV. Fig. 5.) in EF and HI, till the original plane, its parallel, and the directing plane, and along with them the point of ſight O, come into their proper places. Then you will find, that D (Plate IV. Fig. 4.) and F (Plate IV. Fig. 5.) will be the interſection of AB, and G (Plate IV. Fig. 5.) will be its directing point. But OG (Fig. 4.) is ſtill parallel to AB, wherefore G is its vaniſhing point, and DG its indefinite projection, (by Theo. 3.) and F*a* (in Fig. 5.) is ſtill parallel to OG, which is the director of AB; wherefore F*a* is the indefinite projection of AB (Theo. 5.) That *a* found by the interſection of FI with DG (in Fig. 4.), and of Ed with F*a* (in Fig. 5.), is the projection of the interſection of the original lines AB and AC (Fig. 4.), and of AB and AD (Fig. 5.) is obvious. The other conſtruction by the lines AO is the ſame as by the lines AO and CO for finding the points *a* and *c*, Prob. II.

N. B. In Fig. 5. the projections *ad* and *kl* are parallel, their originals, having both of them the ſame director OH, according to Cor. 1. Theor. 5. The ſame may be obſerved in *lm* and *cd*, which have the ſame director OI.

PROBLEM

Problem VII.

Having given the ſame things as in the foregoing problem, to find the projection of any figure in the original plane.

This is done by finding the projections of the ſeveral parts of the figure given, by the foregoing problem.

For example, the projection *klmnp* (Plate V. Fig. 1.) of the pentagon KLMNP is found thus. Drawing OG, OH, OI, OV, parallel to KL, LM, MN, KP reſpectively, the points G, H, I, and V, are vaniſhing points: and KL, LM, MN, being continued, cut the interſections in their interſections Q, R, T; whence drawing QG, RH, TI, are got the projections *l*, *m*, of the points L and M, by their mutual interſections. Then drawing OK and ON, are got the points *k* and *n*. Then drawing *k*V to the vaniſhing point V of KP, is got the indefinite projection of KP. Laſtly, drawing OP is got the point *p*.

The projections of curve-lined figures are to be got by finding the projections of ſeveral of their points, and afterwards joining them neatly by hand. Thus (Plate V. Fig. 2.) DE being the interſection, and VF the vaniſhing line, and O the point of ſight, and ABC an original circle, placed as in the foregoing problem, the projection *a* of any point A may be found by drawing at pleaſure AD, and OV parallel to it; then drawing DV and OA meeting in the point ſought *a*, according to the conſtruction in Problem VI. D being the interſection, and V the vaniſhing point of the line AD, and the ſeveral lines AD being drawn parallel to one another, the ſame vaniſhing point V may ſerve for them all.

Or VF (Plate V. Fig. 3.) being the directing line brought into the picture, the reſt remaining as before, drawing at pleaſure AD cutting DE and VE in D and V; then drawing OV and D*a* parallel to it, the projection *a* is got by drawing OA cutting D*a* in *a*. And the ſame point V being uſed for all the points A, all the lines D*a* will be parallel to one another, and to the ſame line OV.

Problem VIII.

To find the projection of any figure in a plane parallel to the picture.

The projection being ſimilar to its original, (by Cor. 2. Theo. 4.) this is done by making an exact copy of the original figure; making the homologous ſides in the proportion explained in Cor. 3. of the ſame Theorem.

Problem IX.

Having given the vaniſhing line of a plane, its center and diſtance, and the projection of a line in that plane; to find the projection of another line in that plane, making a given angle with the former.

Let the plane HFIE (Plate V. Fig. 4.) repreſent the picture, which raiſe up and place the plane EIO parallel to the original plane HBFH. Now if we ſuppoſe O to be the point of ſight, EI will repreſent the vaniſhing line. Let *ab* be the given projection; it is required to draw *ac*, ſo that the original of the angle *bac* may be equal to a given angle. Continue *ab* to its vaniſhing point G, draw GO and OI, making GOI equal to the given angle, and cutting the vaniſhing line in I; then draw I*ca*, which will be the line ſought.

The reaſon of this conſtruction is very eaſy to be underſtood; for the plane EOI being parallel to the original plane, it is evident that any two lines, as EO, GO, drawn from the vaniſhing points E, G, to the point of ſight O, will form an angle at that point equal to the angle contained by thoſe original lines, of which E and G are the vaniſhing points.

N. B. If it had been required to make *abc* to repreſent the angle ABC, the angle EOI muſt have been made equal to the complement of the angle ABC to two right angles.

Problem X.

Having given the vaniſhing line of a plane, its center and diſtance, and the projection of one ſide of a triangle of a given ſpecies in that plane; to find the projection of the whole triangle.

The

The projections of the ſides wanting are to be found by the foregoing problem, the angles of the triangle being given. Thus having given the projection *ab*, (ſee the laſt Figure) of the ſide AB of the triangle ABC, the vaniſhing point of the ſide *ac* is found by making the angle IOG equal to the angle CAB, and the vaniſhing point of the ſide *bc* is found by making the angle IOE equal to the complement of the angle CBA to two right angles.

N. B. If the vaniſhing point of the line given *ab*, is out of reach, you may proceed thus: Taking any line DR, PlateVI. Fig. 1. (parallel to the vaniſhing line HG by Theorem 6.) for the interſection, by means of two two lines, H*b*E, I*a*F, drawn at pleaſure through *b* and *a*, find the originals A and B of the points *a* and *b*; (by the converſe of Problem I.) and draw AB. Then on the ſide AB complete the original triangle, and find the projection of the ſides wanting by Problem VII.

Problem XI.

Having given the vaniſhing line of a plane, its center and diſtance, and the projection of one ſide of any figure in that plane, to find the projection of the whole figure.

Divide the whole figure given into triangles, by means of diagonals, and find the projections of theſe triangles one after the other (by the laſt Problem), beginning with thoſe that have the line given for one of their ſides.

Problem XII.

Having given the center and diſtance of the picture, and the vaniſhing line of a plane, to find the vaniſhing point of lines perpendicular to that plane.

Let AB (Plate VI. Fig. 2,) be the vaniſhing line given, and C the center of the picture. Draw CA perpendicular to AB, and CO parallel to it, and equal to the diſtance of the picture. Draw AO and OD perpendicular

dicular to it, cutting CA in D, which will be the vaniſhing point ſought; for raiſe up the triangle AOD perpendicular to the plane of the ſcheme, (which here repreſents the picture) then a plane paſſing though the point of ſight O, and the line AB will be the parallel of the original plane, and the line OD will be perpendicular to it, and conſequently will be the parallel of lines perpendicular to that original plane. Wherefore D is the vaniſhing point of thoſe perpendiculars, (by Def. 17.)

N. B. 1. When the vaniſhing line AB paſſes through the center of the picture, that is, when the original plane is perpendicular to the picture, the point D will be infinitely diſtant, the line OD being parallel to AD, and the projections of the lines perpendicular to the plane propoſed, will all of them be perpendicular to AB, they being to meet the line AC, which is perpendicular to it at an infinite diſtance, and conſequently they will be parallel to one another; which they ought to be upon another account, their originals being all parallel to the picture.

N. B. 2. But when the original plane is parallel to the picture, the diſtance CA will be infinite, and conſequently OA will be parallel to CA, and OD will coincide with OC, making the point D to fall into the center of the picture C, agreeable to Cor. 2. Theo. 3.

N. B. 3. CD is a third proportional to AC and CO, as alſo is AD to AC and AO.

N. B. 4. OD is the diſtance of the vaniſhing point D.

Problem XIII.

Having given the center and diſtance of the picture, to find the vaniſhing line, its center and diſtance, of planes that are perpendicular to thoſe lines that have a certain vaniſhing point.

The ſolution of this problem eaſily follows from that of the laſt, for let C be the center of the picture, and D the vaniſhing point propoſed, (ſee the laſt figure) then it is very evident that in order to determine

the point A, we muſt join the points D C, and draw CO perpendicular to DC, and equal to the diſtance of the picture; then draw DO, and make OA perpendicular to it; draw AD perpendicular to the right line DCA, and it will be the vaniſhing line required, A being its center, (by Theo. 1.) and OA its diſtance.

Problem XIV.

Having given the center and diſtance of the picture, through a given point to draw the vaniſhing line of a plane that is perpendicular to another plane, whoſe vaniſhing line is given, and to find the center and diſtance of that vaniſhing line.

Let AB (the ſame Fig. as before) be the vaniſhing line given, and C the center of the picture, and let E be the point given. Find the vaniſhing point D of lines perpendicular to the original planes of AB (by Prob. XII.): Draw DE, which will be the vaniſhing line ſought; draw CF cutting DE at right angles in F, and F will be the center of the vaniſhing line DE (by Theo. 1.) Make a right-angled triangle, whoſe baſe is CF, and its perpendicular is equal to the diſtance of the picture, and its hypothenuſe will be the diſtance of the vaniſhing line DE (by the Cor. Th. 1.)

Now becauſe the plane, whoſe vaniſhing line is ſought, is perpendicular to the other plane, its vaniſhing line muſt paſs through the vaniſhing point D of lines perpendicular to that other plane, becauſe ſome of thoſe lines are in the plane ſought. Therefore DE is the vaniſhing line ſought.

In the above conſtruction, where DE is ſaid to be the vaniſhing line ſought, it is to be underſtood as the vaniſhing line of a plane perpendicular to that whoſe vaniſhing line AB is given, without having regard to its ſituation with reſpect to the picture, only that it ſhall be perpendicular to the plane of which AB is the vaniſhing line; for were the propoſed

perpendicular

perpendicular plane, when produced, to interſect the picture in a given angle, ſuch reſtriction might render it impoſſible that the plane producing its vaniſhing line, ſhould paſs through both the points D, E.

Corollary 1.

If FC be continued till it cuts the vaniſhing line given in B, B will be the vaniſhing point of lines perpendicular to the original plane of the vaniſhing line DE: For that vaniſhing point is in the line FC, by the conſtruction of Problem XII. and it is in the vaniſhing line given by what has been proved in the preſent Problem.

Corollary 2.

And therefore, if the vaniſhing lines AB and DE meet in G, the points B, D, and G, will be the vaniſhing points of thethree legs of the ſolid angle of a cube, which are perpendicular to one another. And drawing DB, BG, GD, and DB will be the vaniſhing lines of the three planes that contain that ſolid angle.

Corollary 3.

The diſtance of the vaniſhing line DG is equal to the line FP, the point P being the interſection of the line FC, with a circle deſcribed on the diameter DG.

Problem XV.

Having given the center and diſtance of the picture, and the vaniſhing point of the common interſection of two planes that are inclined to one another in a given angle, and the vaniſhing line of one of them; to find the vaniſhing line of the other of them.

Let C (Pl. VI. Fig. 3.) be the center of the picture, BG the given vaniſhing line of one of the planes, and B the vaniſhing point of their common interſection, and H the angle of their inclination to one another. Find the vaniſhing line GD, of planes perpendicular to the lines whoſe

vaniſhing point is B (by Prob. XIII.); let that vaniſhing line cut the vaniſhing line given in G. In GD find the vaniſhing point E of lines making the given angle H, with the lines whoſe vaniſhing point is G (by Prob. IX.); that is, in BCF perpendicular to GFD; take FP equal to the diſtance of the vaniſhing line GD, (found by Prob. XIII.) and draw PG, and PE, making the angle EPG equal to H; draw BE, which will be the vaniſhing line ſought.

For a demonſtration of this conſtruction, raiſe up the planes EPG, BPG, until they meet in P, the point of ſight perpendicular over the center of the picture C; then if we imagine a plane to paſs through P, and making the vaniſhing line BD, the planes BPG, GPD, DPB, will be the parallels of three original planes, whoſe vaniſhing lines are BG, GD, DB; that whoſe vaniſhing line is DG, being perpendicular to the other two (by the conſtruction, becauſe it is perpendicular to their common interſection, whoſe vaniſhing point is B.) Therefore the original planes, whoſe vaniſhing lines are BG and BD, are inclined to one another in the angle EPG, or H; (for the inclination of two planes is always meaſured in a plane perpendicular to their common interſection) therefore BG being the vaniſhing line given, BE is the vaniſhing line ſought.

N. B. The center of the vaniſhing line BE is found by drawing a line perpendicular to it from C, (by Theo. 1.) and then its diſtance is found, as was found the diſtance PF in Prob. XIV.

OTHERWISE,

Having drawn the indefinite right line BCF, make CF equal to a third proportional to BC, and the diſtance of the picture; draw DFG perpendicular to BF; then take FP equal to the quotient ariſing by dividing the ſum of the two ſquares of BC, and the diſtance of the picture, by the right line BF; draw GP, and from the point P draw P E, making

an

an angle GPE equal to the given angle H; which being done, draw BE, and it will be the vaniſhing line required.

Problem XVI.

Having given the center and diſtance of the picture, and vaniſhing line of one face of any ſolid figure propoſed, and the projection of one line in that face; to find the projection of the whole figure.

By means of the projection given, find the projection of the ichnography of the figure propoſed on the plane of that face, whoſe vaniſhing line is given (by Prob. XI.); then (by Prob. XIV.) find the vaniſhing line of the plane of the orthography, and deſcribe the projection of the orthography, by help of the lines already given in the ichnography (by Prob. XI.). Laſtly, by the interſections of the projections of perpendiculars to the ichnography and orthography, will be found the ſeveral points of the projection required.

Otherwise,

Having found the projection of the face whoſe vaniſhing line is given, by means of the projection of the line given, find the vaniſhing lines of the adjacent faces, (by the laſt Problem) and deſcribe their projections by help of the lines given in the projection of the firſt face, and ſo on, till the whole projection ſought is completed.

This is a general deſcription of the method to be uſed in putting any figures propoſed into Perſpective, and by the rules herein delivered, the following examples were actually delineated. Having in theſe few pages done my beſt endeavour to render Dr. Brook Taylor's Perſpective eaſy to be underſtood, I ſhould have here ended; but as ſeveral conſiderable writers upon this ſubject have objected againſt the univerſality of theſe rules, and their holding equally good, when applied to putting of ſome particular ſorts of objects, ſuch as round pillars, &c. into Perſpective,

tive, I hope the following attempt to ſet this matter in a clear light, will not be diſagreeable to the reader.

The appearance of an object and its repreſentation are two very different things, the former being determined by the angle (at the eye) under which it is ſeen; but the latter is the actual figure formed upon the picture, by the pencil, of rays paſſing from every point (in view) of the object to the ſpectator's eye; conſequently, in viewing a picture, if the eye be ſuppoſed to move about from the proper point of ſight deſigned by the painter, it is no wonder that ſome remote parts in the picture ſhould appear diſtorted; and for a very obvious reaſon, namely, becauſe it is very poſſible that the repreſentation of a right line may, upon account of its ſituation with regard to the picture, be much longer than the original line itſelf: but that a right line placed in any poſition whatſoever, can appear longer than itſelf, is abſolutely impoſſible, and contradictory to the very nature of the ſcience of Perſpective. But to render this ſtill plainer by an example, let AB, Pl. 39. Fig. 1. be the plane of the picture, CD an original right line, O the point of ſight; draw OC, OD, and ſuppoſe CD ſo ſituated, that the angle ODC may be a right one: then it is very evident that C*d* is the repreſentation of CD; but becauſe C*d* is the hypothenuſe of the right angled triangle CD*d*, it follows that C*d* muſt be longer than CD, that is, the repreſentation is longer than the original line; but notwithſtanding this, it is certain, that to the ſpectator's eye at O, the repreſentation C*d* can only appear as the tangent DC (OD being radius) of the angle DOC, formed by the rays OD, OC, paſſing from the points C, D, to the eye placed at O. For a ſecond example of this kind, let it be required to draw the repreſentation of a range of columns parallel to the picture. If they are drawn according to the ſtrict rules of Perſpective, then that column which is in the center of the picture will be the leaſt, and conſequently thoſe on each ſide of it will be

larger

larger and larger continually, the farther they are removed from the center of the picture, (if the original columns are all of the ſame diameter.) But to explain this more fully, let PP repreſent the plane of the picture, Pl. 39. Fig. 2. H, G, and I, the centers of the ſections of three columns, whoſe repreſentations we are to find upon the picture; and let E be the place of the ſpectator's eye: draw the tangent E*h*, E*g*, EA, EB, E*i*, and E*k*; then will their interſections with PP determine the repreſentations *cd*, *ab*, *ef*. Now it is very evident that *cd* and *ef* are much longer than *ab*. From whence we may conceive, that the farther any column is removed from the center of the picture, the longer will its repreſentation be; and we may moreover conceive, that this increaſe of the repreſentations of the magnitude of the columns, is owing to the obliquity of the chords *hg*, *ik*, with the picture; which chords will meaſure the apparent widths of the columns, if viewed from the proper point of ſight, and will appear leſs and leſs, the farther they are removed from the perpendicular EG. But if we were to draw the appearances of theſe columns upon the picture, without having regard to any particular point of ſight, but only that the eye be placed at a due diſtance from the picture; then with the perpendicular diſtance of the eye from the plane of the picture, deſcribe an arch of a circle cutting the tangent E*h*, E*g*, E*a*, &c. and the chords of the arches ſo made by theſe tangents, will be the apparent widths of the columns to be repreſented in Perſpective. Thus if E*a* = E*b* be the radius, then will *rs* and *tw* repreſent the perſpective appearances of the widths of the columns I and H reſpectively.

END of the THEORY.

PRACTICAL PERSPECTIVE.

IN this practice of Perſpective, I have thought it convenient, that the vaniſhing line to the *ground* plane, ſhould be called the *horizontal* line, and the *interſection* of the *picture* with that plane, the *ground* line. In order to underſtand this well, I have ſet down the following general definitions.

PLATE XXXVIII. FIG. I.

LH, ED, the *picture*; FM, GK, the *ground* plane.

O, the point of ſight; LVH,, the *horizontal* line.

V, the *center*; the line OV, the *diſtance*.

CI, LH, the horizontal plane; CI, FM, the *directing* plane: AB, an original line; *ab* the perſpective repreſentation of it; P the interſecting point; and Z the directing point.

CHAP. I.

Shewing in what manner any ſuperficial figure may be put in Perſpective.

The diſtance the ſpectator ſhould ſtand from the *picture* (or what is meant by the point of *diſtance*) is very eſſential; for on a right choice of it depends the agreeable appearance of the objects. If it is placed very near, the repreſentation of ſome objects will be greater than their

 originals,

originals, as *a*, *b*, (in Fig. 2. Plate 38.) but this muſt be left to the diſcretion of the artiſt, which experience will improve.

EXAMPLE I.

PLATE VII. Fig. 1.

(Is a method made uſe of by an ingenious author, as the neareſt diſtance.)

ABCD, is the *picture*, G, the *centre*, CD, the interſecting or *ground* line. Through the center, draw HL parallel to CD, for the *horizontal* line, on G raiſe the perpendicular GO, equal to GD, or GC, let O be the point the ſpectator's eye is ſuppoſed to be.

EXAMPLE II.

PLATE VII. Fig. 2.

To find the repreſentation of a line given.

The picture being prepared as above, HL, for the *horizontal* line, C, the *center*, O, the point of *ſight*, DB, the *ground* line, and AB, the given line; touching the ground line in B, make OH parallel to AB, interſecting the *horizontal* line at H, for the vaniſhing point of AB; draw BH, and from A draw the ray OA, interſecting HB at *a*; join *a*B for the Perſpective repreſentation of the original AB.

Fig. 3.

The original AB is continued to the *ground* line at *d*, the vaniſhing point H, is found as in the foregoing Ex. The rays BO, AO, cutting the line *d*H in *ba*, gives *ab* for the repreſentation of AB, and as deep in the *picture* as AB is from the ground line.

EXAMPLE III.

PLATE VIII. Fig. 1.

In this example the *original* line DB is perpendicular to the *picture*, HL, the *horizontal* line, C, the *center*, GF, the ground line, and OC, the

diſtance; continue DB to the *ground* line at *e*, and draw at pleaſure two lines, B2, B1, paralſel to each other, cutting GF at 1 and 2, make OE parallel to any one of them, draw 2E, 1E, then draw *e*C, interſecting them at *bd*, join *bd* for the repreſentation of DB. For the line A, make OC parallel to it, then C is the vaniſhing point of A, and DB.

EXAMPLE IV.

To find the repreſentation of a given Triangle.

Let BAD be an original triangle, HL, the *horizontal* line, GF, the *ground* line, OC, the diſtance, produce the ſides AB, BD, to the ground line at 1 and 2. Make OH parallel to AB, and OL parallel to BD, cutting the horizontal line at H and L the *vaniſhing* points, for AB, and BD, draw 1L, 2H, interſecting each other in *b*, draw the rays AO, DO, cutting the lines 1L, 2H, in *a*, and *d*, join *abd* for the repreſentation of BAD. Note AD, is parallel to the ground line: ſo is *ad*.

EXAMPLE V.

PLATE VIII. Fig. 3.

Having an original Square given to find its Perſpective Repreſentation.

Let LH be the *horizontal* line, GF, the ground line, O, the point of ſight, ABDE, the original ſquare, the ſides AD, BE, being perpendicular to GF, and OC, parallel to BE, cutting the horizontal line at C; for the vaniſhing point of the ſides AD, BE, and centre of the picture, continue AD, BE, till they interſect GF, at 1, and 2, draw 1C, 2C, then produce the diagonal DB, to the ground line at 3, and make OH parallel to DB, cutting HL, at H, H is the vaniſhing point of DB, draw 3H, cutting 1C, 2C, at *b*, and *d*, make *ba*, *de*, parallel to GF, (their original being ſo) cutting 1C, 2C, in the points *a*, *e*, join the points *a b d e*, for the Perſpective Repreſentation of ABDE.

EXAMPLE

EXAMPLE VI.

PLATE VIII. Fig. 4.

To give the repreſentation of a ſquare, that lies obliquely to the picture.

Every thing being prepared as in the foregoing example, continue the ſides BA, and ED, of the original ſquare, at the *ground* line GF; to the points 1, 2, 3, 4, make OL parallel to AB, BL, and OH, parallel to BA, ED, draw the lines 4H, 3H, 1L, 2L, interſecting each other at the points, *a*, *b*, *e*, *d*, the *Perſpective* of A, B, D, E.

EXAMPLE VII.

PLATE IX. Fig. 1.

To give the repreſentation of a Pentagon, from an original given.

HL, the *horizontal* line, O, the point of *ſight*, GF, the *ground* line, A, B, C, D, E, the *original*, N, P, Q, M, the interſecting points on the *ground* line of the ſides AB, CB, DC, EA; L, the vaniſhing point of the ſide CB, H, of the ſide AB, K, of the ſide EA, and I of the ſide CD. Draw PH, QL, interſecting each other at *b*, and the lines KM, IN, interſects QL, PH, in *c*, and *a*; draw the rays DO, EQ, interſecting NI, MK, in *d* and *e*, join theſe points to finiſh the projection required.

Figures containing any number of ſides are put in perſpective in the ſame manner.

EXAMPLE VIII.

PLATE IX. Fig. 2.

To find the Perſpective of a circle or any given curved line.

HL, the *horizontal* line, EF, the *ground* line, O, the point of *ſight*, and A, B, C, an *original* circle given, D its *center*. Draw at pleaſure from the point B a line interſecting the ground line at F, and as many through the circle (parallel to BF) as you pleaſe; make OK parallel to BF, then is K, the vaniſhing point to them all. The point *b* of the

original B is found thus, draw FK, and from B draw the ray BO, interſecting FK at *b* the point ſought, the point *a* is found by drawing EK, and AO cutting EL at the point *a*, the points 1, 2, 3, and as many more are found in the ſame manner; the ray DO gives on the line 1K, the point *d* for the center. Theſe points muſt be joined artfully for the Perſpective repreſentation.

EXAMPLE IX.

PLATE X. Fig. 1.

To put an Octagon in Perſpective.

HL, the horizontal line, O, the point of ſight, and GD, the ground line, AB, the original, C, the center, M, N, the interſecting points of the lines MK, IN; H L 1 2, vaniſhing points: viz. 1 for the ſide B, 2 for the ſide A, L, for MK, and H, of IN. Draw NH, ML, and, through their interſection, draw a line 5 6 C, and 4 3 parallel to MN; from L, through the point 3, draw a line cutting 6, 5, at 5, then draw 4L cutting 5, 6, at 6; draw 3, 1, gives 8 on ML, and 4, 2, gives 7, on NH, then a line from 2, through 3, gives *a*, on NL, and a line from 1, through 4, gives *b*, on ML, and finiſhes the octagon.

The circle, (Fig. 3.) is put in Perſpective by the ſame points and manner.

EXAMPLE X.

PLATE XI. Fig. 3.

To give a true Repreſentation of a Circle in any part of the Picture, by having only a Line given as a Diameter.

Prepare the picture in this manner. Let AB be the *horizontal* line, OC, the point of *diſtance*, for convenience to be transferred on the line AB, in A, and B; C, the *center*. Set one foot of the compaſſes in the point B, with the opening BO, deſcribe the arch OL; and from AO, the arch OV; let 3, 7, be a diameter given, through the mid-

dle

dle 9, draw C51, at pleaſure, from A, and B, draw lines through the point 9, at pleaſure. To find the points 1, 2, 4, 5, 6, and 8; from B, through 7, draw a line, interſecting the line 51, at 1; from 3 to B, gives the point 5; from V, through 7, a line drawn cuts the line A, 9, at the point 8, and from 3, to V, gives the point 4, on the ſame line; from L, through 3, a line cuts B9 at the point 2, and another from 7, to L, gives the point 6.

This method may ſerve for an octagon, a ſquare, and many other uſes, and is the readieſt way to repreſent any circles within each other.

CHAP. II.

Practical Perſpective of Solid Bodies.

EXAMPLE XI.

PLATE XII. Fig. 1.

To give the Repreſentation of a Cube from an Original given.

Let HL be the *horizontal* line, C, the *center*, OC, the *diſtance*, and H, *vaniſhing* point of CB, and ABCD the plan of the original cube, find the Perſpective ſquare, *a*, *b*, *c*, *d*, of ABCD, (by Ex. V. Plate VIII. Fig. 3.) at point 2 on the ground line raiſe a perpendicular 2*e* equal to any height given, on every point *a b c d* raiſe perpendiculars at pleaſure, from the point *e* of the line 2*e*, draw a line to the center C, cutting the perpendiculars *b d*, at 6 and 7, from the points 6 and 7, draw lines parallel to *a b*, cutting the perpendiculars *a* and *c* at 5, 4.

EXAMPLE XII.

PLATE XII. Fig. 2.

To find the Perſpective Repreſentation of a Pyramid, the Ichnography, or Ground Plan CDEF *being given, whoſe Poſition is oblique to the Picture.*

Let HL be the *horizontal* line, S the *center*, O point of *ſight*,

L *vaniſhing point* of the ſides CD, EF, H vaniſhing point of CE and FD, the line AB on the *ground* line is a given height. Find the *Perſpective* plan *c d e f* (by Ex. VI. Plate VIII. Fig. 4.) draw *ed*, *cf*, to find the center *a* of the Perſpective plan, on *a* raiſe a perpendicular at pleaſure; from L through the center *a* draw a line to the ground line, at that interſection ſet the given line AB, draw BL cutting the perpendicular *ab*, at the point *b*, join *bc*, *bd*, *be*, *bf*, and compleat your pyramid.

EXAMPLE XIII.

PLATE XII. Fig. 3.

To put a Cube in Perſpective ſtanding obliquely to the Picture.

Every thing ſtanding as in the foregoing example, the Perſpective ground plan *a b c d*, of the original ABCD, is found (by Ex. VI. Plate VIII. Fig. 4.) at the point 1 ſet a line 1F for the given height, on every point, *a*, *b*, *c*, *d*, raiſe perpendiculars at pleaſure, draw FL, cutting the perpendiculars *a*, *d*, at 5, 6, from H, through 5 and 6, draw lines, cutting the perpendiculars, *b*, *c*, at the points 8 and 7, then is your ſolid figure finiſhed.

EXAMPLE XIV.

PLATE XIII. Fig. 1.

To put a Chair in Perſpective from a given original Plan, whoſe Poſition is parallel to the Picture.

Let ML be the *horizontal* line, N, the *center* and vaniſhing point of the ſides AC, and BD, of the original plan X. M, the vaniſhing point of the diagonal DA, the ſmall ſquares on the original plan are the plans of the legs; find the Perſpective plan *a*, *b*, *c*, *d*, (Ex. V. Plate VIII. Fig. 3.) from the points 2, 3, on the ground line 1J, draw 3N, 2N, cutting KM; and there determine the little ſquare, whoſe parallel ſide is 5*a*, continue a line to 6 it will give *b*6, and ſo finiſh the other ſquares; at *d*

and

and *c*, and on every point *a* 5, 6, *&c.* of the Perſpective plan, raiſe perpendiculars at pleaſure. At the point 4 on the ground line 1 J, ſet the perpendicular RS, with the heights of every member of the chair, marked *z*, *y*, *x*, *g*, 8, 7, and from every one of the points draw lines to the point N, cutting the line *l b*, in *l*, *u*, *v*, *t*, *e*, *f*, *h*, from *f* draw *f* U T, cutting the perpendicular *a* at T, draw TN cutting the line *ck*, at *k*, draw *b k*, parallel to U, T, then will three ſides of the chair be compleated; a line from *f* to N finiſhes the back. At the point U draw a line to the point N to croſs the uprights, in thoſe little lines U and *k*; To find the rail *s*, draw 7N, cutting *lb* at *h*, and from *h*, draw a ſmall line acroſs the perpendicular on the point 6, parallel to *b a*; and from that interſection draw a line through N, to the leg *d*, which will give the upper ſide of the rail *s*; the ſame muſt be done for the other ſide from the line under *h*: it is eaſy to imagine the rails Q, and P, only they don't go acroſs the uprights but half way, as the line RS, on the point 4 is in the ſame plane with the back of the chair. It finiſhes only that ſide, the other ſides are found in the ſame manner as the cube, (Fig. 1. Plate XII.)

EXAMPLE XV.

PLATE XIII. Fig. 2.

To give the Repreſentation of a ſquare Table.

Every thing ſtanding as in the foregoing example, let ABCD, of fig. 2. be the ground plan of the table, and the little ſquares that of the legs; 1, 7, 6, 2, their interſecting points on the ground line. The line on the point 2, the *height* of the table with the parts, markt 5, 4, 3: *a*, *b*, *c*, *d*, the *Perſpective* plan of A, B, C, D, *aq*, *pc*, the thickneſs of the feet, the rail *f* is carried parallel to FJ, cutting the perpendicular *q*H, through that interſection, and *f* draw lines to point N interſecting

the back feet at *b*, and from thence a line parallel to *a*, *c*, through the leg *b*; every line that ſerves to ſhew the thickneſs of the legs are contrived in that manner, G, H, E, F, repreſents the top.

The ſcheme will be ſufficient without any farther explanation.

EXAMPLE XVI.

PLATE XI. Fig. 1.

To give the Perſpective Repreſentation of a Cylinder, (ſtanding perpendicular on the Ground) by only a Line given in the Picture, as the Diameter of a Circle.

Let OH be the *horizontal* line, H, the point of *diſtance* laid on the horizon, O, the *center*, and V a *point to determine the circle*. Let *ab*, be the line, the circle *a*, B, *b*, is found, (by Ex. X. Plate XI. Fig. 3.) On the points *a*5, B6, *b*, *i*7, *c*8, and on the center, raiſe perpendiculars at pleaſure, ſuppoſe BD, a height given, draw DO, cutting the lines, *x*, *c*, at *e*, and *f*, and through *e* draw a line parallel to the horizontal line cutting *a*, *b*, at *g*, *h*, from V through *g* draw a line, which gives the point 1, on the perpendicular line 5, and a line from *h* to V gives the point 3 on the line 7, from 3 draw a line parallel to *g h*, it gives the point 4 on the perpendicular 8, and through 1 a line parallel to *g h*, gives the point 2 on the line 6, then join thoſe points for the circle, on the point *i*, raiſe a perpendicular to interſect the circle above at *r*, and at point *s*, erect another to give the point *w*, the ſtep on which ſtands the cylinder is to ſhow the perſpective better. The line *i r* is farther from *b*, towards the center O, as the line *sv* is from *a*, nearer the ground line.

This repreſentation, which is not half, is as much as we can ſee of a cylinder, or any round body from that point of ſight.

EXAMPLE

EXAMPLE XVII.

PLATE XI. Fig. 3.

To ſhew the Perſpective of a Cylindrical Body lying on the Ground at Right Angle with the Picture.

The picture having every thing ſtanding as in the foregoing example, the line *a*C vaniſhing at O, is the axis running through the cylinder. Every circle on the body are parallel with the *picture*, and are ſtruck with compaſſes; draw B*b*, to the point O. On the *ground* line BG, ſet the meaſures for the diſtances between the circles, and length of the body. To find the circle *c d e*, on the line BG, let the point A be the diſtance for one circle: draw AH, which gives the point *e* on B*b*; at that point raiſe a perpendicular through the line *a*C, and *a* is the center for that circle. If the object ſhould repreſent a cannon, as they diminiſh at the muzle, let C be the ſmall end, draw a perpendicular through C, cutting B*b* in *b*: let 3, 4, on the line *e*C, be equal to the ſmall end wanted; draw from 3 and 4 lines to O, cutting the line C*b*, in 1 and 2, ſet one foot of the compaſſes in C as center, with the opening C1, or C2, deſcribe a circle; divide your circles equally in as many parts as you pleaſe, and join them together with ſtraight lines, as *c*2, *d*R, &c.

EXAMPLE XVIII.

PLATE XIV.

To put any kind of Wheels in Perſpective.

In this example is the repreſentation of a water-wheel, with coggs. HL the *horizontal* line, H the point of ſight, E point of diſtance laid on the horizon, CM the *ground line*, VHV the *vaniſhing line* of the plane BEGI, EF or 5D the thickneſs; from the point 5 draw a line 5G to the *center* H, and from C draw CL, to terminate the line 5G, at G; and from K, the center of the circle to the point L, given *e*, on BG, from the line BG, finiſh the perſpective

ſquare BGIE, draw the diagonals GE, 5I, interſecting at O, draw HO*a*, and *e o f*, then is the ſquare prepared to draw the circles by the points VV, as has been ſhewn in (Plate XI. Fig. 3.) then on the point D for the thickneſs, draw DF, parallel to 5E, and with that line finiſh another ſquare as is repreſented by the dotted line, and draw from each corner diagonals, as in the firſt ſquare: when your firſt circle *e d t* is finiſhed, ſet off any given height for the coggs, on 5E as 6, draw 6H, cutting *e f* in the point, to form the circle of the wheel, and ſo on for the next. To divide the wheel, draw a quarter of a circle as K, and divide that in as many parts as you pleaſe, it is here in 4; from every one of theſe points draw perpendiculars to the ground line, and from their interſection with it draw lines to the point of diſtance L, cutting BG, and then draw perpendiculars through one half of the perſpective circle, and from thoſe points draw lines through the center of the wheel acroſs to find the others. For example, to find the points *b* and *f*, from B draw a perpendicular, cutting the circle in *b* and *f*, from *f* thro' the point O, draw a line croſs the circle for the point *k*. For the thickneſs of the coggs, (ſuppoſing them in the original to diminiſh to the center) from the point *n* of the cogg *f*, draw a line thro' the point O acroſs the circle, gives the point *l*. Thoſe lines are drawn to the line VHV, and gives V, T, for vaniſhing points of the ſides *f*, *n*, by this mean every circle is divided, from every point *f*, *n*, draw lines parallel to the horizon, interſecting the outward circle at *p*, and from the vaniſhing point V of *n*, draw *p*V, from *d* draw a line parallel to *n*, *p*, cutting *p*V, at *c*, then is *np*, *dc*, a compleat ſide of one of the coggs. To find them without uſing vaniſhing points, as it is often very troubleſome to get, make the circle *c*, then from *d* the parallel line cutting that circle in *c*, join *c p*, the eight ſpokes are the principal lines made uſe here for the circle, and as it is ſuppoſed muſt paſs through the middle of the thickneſs of the wheel, proceed thus, divide 5, D, in P, and a line drawn from P, to the point of diſtance on the line VHV, will give the line through the middle of

the

the ſpoke; but to do it without that point of diſtance, draw the line *m* parallel to the *horizon*, divide that in two equal parts at *m*, draw *m p*, proceed for every other, join the extremity of thoſe croſs lines in a circle, to ſhew the inſide thickneſs of the wheel.

CHAP. III.

Relating to put in Perſpective any Object or Objects in general.

EXAMPLE XIX.

How to put Doors in Perſpective, and to deſcribe Pannels thereon.

Let Plate V. Fig. 1. repreſent the inſide of a room, HL *the horizontal* line, C the *center*, OC the *diſtance*, V a point to find the ſemicircle on the floor, (deſcribed by the door opening) UR a line equal to the breadth of the door, continue U to the *center* C, draw a line from the point R to the point of diſtance (removed on the horizontal line, omitted on the plate for want of room) interſecting UC at X; let UII, be a given height, draw II C, and on X raiſe a perpendicular cutting II C, at III, then is UX, II, III, the aperture of the door; draw RC, and from X a line parallel to UR, cutting RC at T, draw XR for a diagonal, X the center, from V through T draw a line cutting XR at Y, join UYT for a quarter of a circle. The other quarter, if need be, is found, by finiſhing the ſquare XT*b*Z, and drawing the diagonal XZ, and from C and Y draw a line cutting XZ at *y*; join T*yb* for the other quarter; chuſe any point on the ſemicircle for the opening, as F, draw a line from that point through X, to the horizontal line at H, for the vaniſhing point of lines parallel to that, and from H, through III, draw a line at pleaſure, on F, raiſe a perpendicular cutting X, III, at *a*, then III *a*, XF repreſents a door opened. To form the pannels on the door, draw FE parallel to the horizon; ſet the foot of the compaſſes in H, and with the

opening HO draw the arch OL on the horizontal line, and L is the point of *diſtance*, and H the *vaning point* of the door; from the point L, through X, draw a line, cutting FE at E, then EF is equal to the original of FX1, ſet on FE, the meaſures 1, 2, for the width of the pannels, and from them points draw lines to the point L, cutting XF at 3, and 4, from any points on F*a*. For the height, draw lines to their vaniſhing point H, cutting the perpendicular raiſed on 3, and 4, and terminating the pannels A and B.

The door, Fig. 2. at the end of the room is found in the ſame manner; the line KN the breadth, and KMPN a ſquare made of that line, K is the center, or hinge of the door, with the point V on the horizon; find the point *p*, and finiſh the quarter of the circle, KW height of the door, the dotted lines from K and W vaniſhes to the ſame point on the horizon.

Fig. 3. is the repreſentation of a trap-door on the floor. G9 the breadth, and G D 7 9, a ſquare in perſpective of the aperture, the door, D Q 6 7, is found the ſame as Fig. 1. or Fig. 2. KIDG is a dotted ſquare, equal the aperture, *l* O the vaniſhing line for it, C the center, OC the diſtance, and the points J and L, to find the circle, then proceed as in the other Example.

EXAMPLE XX.

PLATE XVI. Fig. 1.

How to give the Perſpective Repreſentation of Rooms, Doors, &c. *The Point of Diſtance is ſet on the Horizontal Line in this and the following Examples.*

Let HL be the *horizontal* line, S the *center*, L point of *diſtance*, AU breadth and depth of the room, AW the height, draw AS, US, interſect US by the line AL in C, draw CB parallel to AU, then will ABCU repreſent the floor in perſpective. To finiſh the ſides, proceed as with a cube.

To ſet a door on the ſide of the room. On the ground line AU, let *a*U be the width of the door, and U *b* the depth it muſt be in the room, draw,

*b*L cut-

*b*L cutting US at F, from F draw FE at pleaſure parallel to AU, draw *a*S cutting FE at E, and the line EL will cut US at *e*, on *e* and F raiſe perpendiculars to make the door, the ſemicircle *e*EG and door is found as in the foregoing example, K is the vaniſhing point of the door I, H the point of the door N, *t* of the door T, and I of the door M. The projection of the chimney-piece is found as the door, by ſetting on the ground line the meaſures, and proceed as with the door. 1, 2, repreſents the back part of an upper room, *r* another behind it, and T the door opening within it; E the under room, R a room and pavement behind that, V vaniſhing point of the ſtairs.

EXAMPLE XXI.

PLATE XVI. Fig. 2.

How to put Windows in a Room from any given Meaſurement.

Let HL be the *horizontal* line, A*m* the *ground* line, L point of *diſtance*, H the point of ſight, the points *m*, *n*, *p*, *q* the breadth of the panes and frame, and B, C, D, the height of them; lines from *m*, *n*, &c. to the point L, gives on AH, the points 1, 2, 3, 4; on theſe points raiſe perpendiculars; the perpendiculars 1, 4, gives *a*W on the line QH, for the ſquare of the window-frame. To give the thickneſs, and repreſent the ſides, take OQ for the thickneſs of the wall, draw OH from *a* draw a line parallel to the horizontal line, interſecting OH; then *k k* is the upper thickneſs, and where the line *a* interſects OH, draw a line parallel to it; *a*1 gives you the ſide: where the perpendiculars 1, 2, 3, interſect the under frame of the window, draw parallels to HL, acroſs the window-ſeat, interſecting PH; at thoſe points raiſe perpendiculars for the uprights of the ſaſhes, and from BCD, draw lines to the points H to finiſh the ſquares, continue *a*WQ to interſect the line *b* 5, (at the end of the room) at 5, draw 5*l* parallel to HL, for the top of the window; do the ſame to get the bottom, as is ſhewn by the dotted lines 1*t*, draw *xt*, parallel to *lf*, and from *m*, *n*, *p*, *q*, draw lines to H, cutting *b b*, at *e d c*. To

find

find the thickneſs *g*, *f*, continue O*k*, to cut the line 5*b* at 6, and from that point draw a line parallel to *l f*, draw *f g* to the center H, interſecting 6*g* at *g*, from that point draw a line parallel to *f*, *e*, for one ſide of the window; and draw zH, interſecting that line; from that interſection draw a line parallel to *x z*, for the thickneſs of the bottom. The croſs barrs are found by drawing lines from the points 7, 8, 9, to W, and from their interſections on the line *g*, draw lines parallel to *x z*.

EXAMPLE XXII.

PLATE XVII. Fig. 2.

To put a Tuſcan Pedeſtal in Perſpective.

On the *ground* line AK, raiſe a perpendicular *a c*, for the height of the members, as 1, 2, 3, 4 *b c*, for the projection of thoſe members take 4*e*, 3*d*, *&c.* On any point of the ground line, raiſe the perpendicular AC; on it ſet the points 1, 2, 3, 4 B C, equal to thoſe in *a c*, as A*i* the height of the plinth, *&c.* At 1 draw GF parallel to AK, 1G or 1F is equal to half the width of the plinth, from G and F draw lines to the center H. On the ground line ſet the depth (equal to the line G F) from G to K, draw K to the *point of diſtance* I, cutting G H, at the point *l*, raiſe a perpendicular, and finiſh the plinth; from each corner draw the diagonals, F*h*, GL, interſecting each other in O, the center of that ſquare; ſet the different projections from the line *a* to *d*, and put them on from 1 towards D, draw 1H, and from the firſt point on 1D draw a line to I interſecting 1H, at the interſecting point draw a line parallel to GF, cutting the diagonals GF, and from thoſe interſections draw lines to the point H, interſecting the diagonal at *h*, on each point raiſe perpendiculars, for the height of the fillet, then from the point 2, the height, draw a line 2H, cutting the perpendicular on the line 1H, for its proper height; from that interſection draw a line parallel to GF, interſecting that perpendicular, and them on the diagonals G F, and finiſh the fillet in the

ſame

ſame manner as the plinth; the other members are laid on in the ſame manner. For the capital of the pedeſtal, proceed as for the baſe. As the flat band BC projects fluſh with the plinth, from B to E lay the profile of the members, draw BH, interſect it by EL, and finiſh every member before you lay another on it.

EXAMPLE XXIII.

PLATE XVIII. Fig. 2.

How to lay Members of Architecture on one another.

Let ABCD be a ſquare plan in Perſpective, where on it another is to be put on equally every way, GP *horizontal* line, G *center*, P *diſtance*, *r x* is the *diſtance* it is to be laid on; draw the diagonals AC, BD, where *r*P interſects *x*G, draw the line *a d* parallel to AD, cutting the diagonals at *a* and *d*, from thoſe points draw lines to G, cutting the diagonals B and C at the points *b c*, then will the plan *a b c d* be placed equally on ABCD.

EXAMPLE XXIV.

PLATE XVII. Fig. 1.

In this Example is the Perſpective of the Tuſcan Entablature.

Let X repreſent the original, on the line AD are ſet the height of the parts, *viz.* 1, 2, 3, 4, *&c.* and from thoſe as AB, 1*b*, 2*c*, 3*e*, *&c.* are the diſtances they are from that line, (this figure is put on the plate upſide down for want of room) LH, the *horizontal* line, H the point of ſight, and L the point of *diſtance*, laid on the horizontal line. Let FE be equal to the whole width of that member, divide it into two equal parts at *a*, let fall the perpendicular *ad*; on that line ſet the meaſures from AD, 1, 2, 3, *&c.* on *a*F ſet their projections from the line AD, as *b*, *e*, *f*, *g*, *&c.* with the line FE make a perſpective ſquare plan, draw diagonals acroſs, from *a* draw a line to H, from *b* draw *b*L cutting *a*H at *n*, through *n* draw a line parallel to FE cutting the *diagonals* at M M, draw MH till it cuts the diagonal;

on

on *n*MM raiſe perpendiculars. From the point 1 on *ad* draw a line to H, cutting the perpendicular *n*, and from that point draw a line parallel to FE, cutting the perpendicular M M, and finiſh that member as if it was a plinth; then on the corners at F and E draw the quarter round, and proceed the ſame for the reſt, obſerving that every member lies properly on the diagonal of the preceding.

EXAMPLE XXV.

In this Example is the Perſpective of the Tuſcan Baſe.

Let X in Plate XVIII. Fig. 1. be an original ſection made as in Plate XVII. Fig. 1. But to give the proper repreſentation of the ſwell of the Torus, I have divided the original in two equal parts, horizontally as marked 2, and vertically marked *x*; and where the line *x* cuts the Torus, lines are drawn parallel to *x a* againſt *a*5, in the points 1 and 3, to expreſs the ſwell. The baſe is ſuppoſed parallel with the picture, and the plinth BD, whoſe height is AP, is put in Perſpective in the ſame manner as a ſquare; SL the *horizontal* line, S the point of ſight, and L for the *diſtance*; on the ſurface of the plinth, draw diagonals. Their interſection *o* is the center of the baſe, through that point *o* draw a line parallel to DB, for a diameter, and from the S through *o* draw a line for the other, and given the point A on BD, with the point V make the circles on the plinth, as has been ſhewn in Plate XI. Fig. 3. On A5 ſet the height of every member, as 1, 2, 3, *&c.* on AJ are the meaſures for their projections. The point *c*, for one perſpective circle, is found by the line JL, cutting AS in that point; the other points on that line are found in the ſame manner: thoſe points are afterwards carried on one of the ſemi-diameters, by continuing a line from thoſe points to the point of diſtance interſecting the ſemi-diameter, then with a pair of compaſſes carry them over to the other, as *a b c*, then from V and thoſe points finiſh your circles, on the center *o* raiſe the perpendicular K*o*, and through every point 1, 2, 3,

&c.

&c. on the line A5, draw lines to the point S, cutting K*o* in 1, 2, 3, &c. On that part of the circle you intend for a ſection, draw a line through the center K*o*, to the horizontal line; and the interſection will be the vaniſhing point, from that point draw lines through every point on K*o*, cutting the perpendiculars on the points for the circle of that ſection. For Example: Suppoſe for the ſection on the line AH, S is the *vaniſhing point* for it, on every point *a b c*, on AS raiſe perpendiculars, and from S through every diviſion on K*o* draw lines, cutting them. The firſt point on A marked 2, for the middle or greateſt ſwell of the Torus; on the ſecond, in two places to expreſs the turning of the round; on the third, at *b* 4 and 5, the point *b* joining the plinth, and 4 where the fillet begins, 4 and 5 the height of the fillet, on *c* at *d* the ſhaft begins, do the ſame on as many parts of the circle as you think neceſſary to compleat your figure. Then join artfully the points 4 4, 3 3, 2 2, &c. in curves. R is part of a ſection behind that on the diagonal B, *a* is part of another; to finiſh this figure to a great nicety, girt the ſections with as many of thoſe curves as will fill the ſpaces.

When the learner has accuſtomed himſelf in viewing a true repreſentation, he may with only the circles compleat the Torus.

EXAMPLE XXVI.

PLATE XVIII. Fig. 3.

In this Example is the Perſpective of the Tuſcan Capital.

The line AJD is made uſe as a *ground* line, HL the *horizontal* line, L the *center* of the picture, H point of *diſtance*, and V one point for the circle, CD the width of the abacus, J the middle of it, *a b d g* the projection laid on from J towards D, J*r* a line for the height of the parts, as 1, 2, 3 &c. KO the center of the column, ABDC an original ſection; proceed with this exactly as with the foregoing Example.

EXAMPLE XXVII.

PLATE XIX. Fig. 3.

In this Example is the Perſpective of a double Croſs.

Let LH be the *horizontal* line, AD the *ground* line, L the *center*, H the point of *diſtance*, ABCD the extent of the croſs bar, BC the thickneſs; draw AL, BL, CL, DL, and from A draw AH, gives the points *b c* E; through theſe points draw lines parallel to AD for *a g* on AL, and *d* on DL; on the point B ſet the perpendicular BG, with the height of the parts required; then on the points *b c e f*, plan of the upright poſt, raiſe perpendiculars at pleaſure, and through the points on BG, draw GL, KL, IL, cutting *b o* at *q r*, draw through the point *r* a line parallel to the ground line, croſs the perpendicular lines *c b* as *m r s t*, and another parallel to it through *q*; IK, K*n*, is the end of one of the croſs bars, draw KL and IL; perpendiculars on *a g d* F will terminate the points *m*Q*t*, &c. where the line IQ interſects *f p*, draw a line parallel to *m t*, from *t m* draw lines to the point L, a perpendicular on *g* will finiſh the ſide *m*L.

Plate XIX. Fig. 2. is the repreſentation of a bureau with the flap open. *h, i, k*, is the circle deſcribed by the flap opening, PL the *horizontal* line, L the *center*, O, O, X, the *vaniſhing* line to BA*i*, O O two points to contrive the circle, X the point of diſtance on OLX, A*k*M the flap, A the point it turns on; proceed in this as in Plate XV.

EXAMPLE XXVIII.

PLATE XIX. Fig. 1.

To find the Middle or Center of any Building, &c.

Let HV be the *horizontal* line, V the *center*, H the point of *diſtance*, and EC one ſide of the building, through E draw HA, to the ground line AG, divide AC in two equal parts at F, draw FH, cutting CE at D (the middle of EC) raiſe the perpendicular DL, cutting RK at L, draw a line parallel to

AG, interſecting MV, divide that line in the middle, and deſcribe the ſemicircle LQ from V, and through the center of that ſemicircle draw a line, it will give the point Q on it, which is the center of the arch MNR, as required.

EXAMPLE XXIX.

PLATE XX.

In this example the eye is ſo placed in the air as to look down in the building, VUB and RUS are two *vaniſhing* lines at right angle, U is the *center* of the picture, R and S two points of *diſtance*; the line GI parallel to RS, for convenience, muſt be conceived as a *ground* line, ABCD an original arch given, PG is equal the line BC, PA the thickneſs of the pilaſter, PI is half the breadth, and NO the extent of the capital and baſe; interſect PU by GR, to get the point *g*; LM drawn to R, gives on PU, *h*, *l*, *m*, the ſame parts of the original, as by their capitals; from them, lines are drawn parallel to GI, IU terminating the ſide *a*, NU gives *n n* for the projection of the baſe and capital. To turn the perſpective arch, W on the ground line being for its center, draw WU cutting *t t* at *x*; from R and S through *x*, draw lines at pleaſure, for diagonals *x v*, *x y*. To turn the arch, draw Z *t v*, Z *t y*, S *t p*; join *t*, *y*, *p*, *v*, *t*. The ſides of theſe pilaſters are parallel to VUB; the window 1 2, 3 4 is found by drawing 2U, 5U; for the thickneſs, draw 1U, and draw 4 3 parallel to RS, cutting 1U at 3, and from 3 draw a line parallel to VB; finiſhing the bottom of the window, if any depth for the window is required, ſet them on the front line, draw SU, and interſect that line, by another from a given point, to a point of diſtance, as *s*T draw *s t* parallel to VB, then that is the ſide of a ſquare; and if from *t* you draw another to the point of diſtance, cutting SU, the window will be double its breadth: In this ſcheme the pavement being parallel to the *picture*, every ſquare remains the ſame. To find the point of diſtance T, the line VB, and T*s* their interſection is the point required. Cielings are repreſented in the ſame manner. Theſe arches need only be inverted.

CHAP. IV.

To repreſent inclined Planes and Objects.

EXAMPLE XXX.

PLATE XXI. Fig. 1.

To repreſent an inclined Plane, whoſe inclination is given, having one ſide parallel to it.

Let HL be the *horizontal* line, S the *center*, or point of ſight, L the point of *diſtance*; draw SO perpendicular to HL, draw LO parallel to Y the inclined ſurface, cutting SO at O, for the *vaniſhing* point of that plane; through that point, draw VU parallel to HL; ſet one foot of the compaſſes at O, and the other in the point of diſtance L, draw the arch LV; then V is the point of diſtance to it. When the picture is thus prepared; ſet GB as one ſide, on the ground line GF, draw GO, BO, and from V, draw GV, cutting BO at I, draw IK parallel to GB; for the bottom that lies flat on the ground, draw BS; then finiſh it by the perpendicular *1a*. The ſquare *de*, *fg*, Fig. 2. is found the ſame; *de* being given.

EXAMPLE XXXI.

PLATE XXI. Fig. 3.

In this Scheme is the Perſpective of a Priſm that has one Side perpendicular to the Ground, and reſteth on an inclined Plane.

Let HL be the horizontal line, S the center, and the point L the diſtance, Z the original priſm, X the inclined plane, draw SO perpendicular to HL; ſet the priſm Z at the point of diſtance L, and draw LO parallel to the upper ſurface, and LV for the under, cutting VSO at O and V; through theſe points draw lines parallel to HS, and from V and O, with the diſtance L deſcribe the arcs KL, LN; the ſurface of the wedge being parallel with the upper

ſurface

ſurface of the priſm, the point O is the vaniſhing point to both, and V vaniſhing point to the under ſide of the priſm; for the perſpective of the plane X, proceed as in the foregoing example, Fig. 1. Let *n l* on the perſpective plane X be given (parallel to HL) through *n*L, from V draw lines at pleaſure, from N point of diſtance through *l* draw a line cutting *n a* at *a*, make *a b* parallel to *n l*, from *a* and *b* draw *a*O, *b*O either draw a perpendicular on *l* to find the point 1*d*, or the line K*a*, make *d e* parallel *a b*. Fig. 4. is the ground plan of a ſquare (whoſe poſition is oblique to the picture) on an inclined plane, *b d* a line given. From I and K the two points of diſtance, draw lines through *b* and *d*, interſecting each other at *a* and *c*.

EXAMPLE XXXII.

PLATE XXII.

In this Example are Stairs put in Perſpective, ſtanding parallel with the Perſpective.

Make NL the *horizontal* line, S the *center*, N the *diſtance*, I the given inclination, V the vaniſhing point of the inclination found, as by (Ex. XXX. Plate XXI.) Let 3 4 on the ground line be the depth, and FE the height of a ſtep; draw FS, ES and 3N, 4N cutting ES at 2 and 1, there raiſe perpendiculars cutting FS at 5W for the firſt ſtep of the flight K, then draw 5V, WV, the point *d* is found by drawing a perpendicular from W, interſecting 5V at *d*, then draw *d*S interſecting WV, and ſo on for as many ſteps as you pleaſe. The rails of the flight C vaniſhes at V, and the top of the landing place at S; and every ſteps having their ſides perpendicular to the picture, the points *k q p*, &c. are found as the point *d*, the ſtairs N that runs below, are found by drawing a line from V through Y, and proceed as with the firſt ſtep K. X a ray of light, and parallel to the picture, 6 6 *r r* ſhadow of the rail D on the ſtairs.

EXAMPLE

EXAMPLE XXXIII.

PLATE XXIII. Fig. 1.

In this Example is an inclined Chair in Perſpective, and parallel with the Picture.

Let SQ be an *horizontal* line, CD a *ground* line; make USN perpendicular to SQ, S the *center*, Q the *diſtance*, QU a vaniſhing line for the ſide KA, or inclination above the horizon, NU the vaniſhing points, QN the vaniſhing line for the ſide DA, CB, &c. or inclination below the horizon; through the points N and U, draw lines perpendicular to NSU; on them ſet P and R as points of diſtance, P equal to NO, and R equal UO, on C2; 1D thickneſs of the legs; for the plane ABCD draw NCB, NDA, then from P draw a line through D, cutting BC at B, make BA parallel to CD; for the ſide BLKA, being a ſquare, draw BU, AU and BR interſecting AU at K, draw KL parallel to AB. For the ſide ML, CB, draw CU and LN cutting CU at M; the rail *b*5 is parallel to CD, and the ſides vaniſhes at N. The diagonals FG, JP, interſects at O, for one of the rails FP, another &c. The length of this chair is known by the number of diagonals, every two diagonals forming a ſquare, whoſe ſide is equal to BA. Fig. 2. is a chair finiſhed.

EXAMPLE XXXIV.

PLATE XXIV. Fig. 1.

To put an inclined Cube in Perſpective, whoſe Poſition is oblique in the Picture.

Let HL be the *horizontal* line, S the *center*, the perpendicular, SO the *diſtance*, and *a c* a given ſide of a cube in the picture: continue *a c* to the horizontal line at H, draw HO and LO perpendicular to it; then H and L are two vaniſhing points. L for *d e*, H for *b d*, *a c*, *f g*, through L draw CLA perpendicular to HL; then with one foot of the compaſſes in the point L, and with the opening LO, deſcribe the arch OC; make AL equal LO, C is the vaniſhing point of *d f*, *a c*, *b g*, and A of the ſides *a d*, *e f*, *c b*; draw

HA

HA, vaniſhing line for the ſide *a c*, *b d*, &c. draw SB perpendicular to AH. To find the point of diſtance E of that line, divide HA in two equal parts at G; ſet one foot of the compaſſes in G, with the opening GH deſcribe the ſemicircle HEA; through B draw a perpendicular line to HA, cutting the ſemicircle at E; join AE, HE, which is a right angle, as the original. Through the given line *a c*, draw A*ad*, A*cb*, at pleaſure; at the point E ſet off 4 5 *d o*, (that being the angle, the diagonal makes with the ſide of a ſquare;) at *b* and through *b*, draw E*b*D, from D through *c*, draw a line cutting *a d* at *d*, and draw *d*H, interſecting *c b* at *b*, draw *d*C, *b*C, *a*C, and *d*L as a diagonal, cutting *a*C at *e*, draw A*e* at pleaſure, cutting *d*C at *f*, and draw *f* H, interſecting *b*C at *g*, and the cube is finiſhed. If the diagonal *d g* is required, find *k* on CH, as the point D on AH.

EXAMPLE XXXV.

PLATE XXV. Fig. 1.

In this Example in an inclined Chair in Perſpective.

Every line points, &c. ſtanding as in the foregoing; the diagonals are got by E, D and L; *a b*, *c d*, *e f g*, are found as the cube; the back of the chair is three times its width, as appears by the diagonals *d g i* and *k*, which are drawn to the point E.

CHAP. V.

Of finding the Shadows of given Figures.

Shadows are only the projections of given figures on given ſurfaces, by means of given luminous bodies; but to avoid the difficulties that would attend the deſcription of ſhadows, if the magnitude of the luminous body were taken in conſideration, it being ſufficient for practice to regard only the center of it; therefore in theſe examples, a luminous body will be conſidered as a point.

EXAMPLE

EXAMPLE XXXVI.

PLATE XXV Fig. 1.

Let HL be the horizontal line, C the center, and R ray of light, ſuppoſed to come from the ſun, and parallel to the picture; from every corner of the Fig. A at top, draw lines parallel to R, as 1 4, 2 3, and from their ſeats *a* and *b* draw lines parallel to the ground line, interſecting them at 4 and 3, or elſe find only the point 4, as C is the vaniſhing point of the ſide *a b*, 1 2, draw 4C, and at *b* draw a line parallel to the ground line, cutting 4C at 3.

The ſhadow *f e g* of the priſm B is found in this manner, L is the vaniſhing point of the ſide *c d c*, make *f* parallel to the ray R; from the ſeat of *c* at *a*, draw a line parallel HL, interſecting *c f* at *f*, draw *f* L, and from *g* the ſeat of *d*, draw a line parallel to *a f*, cutting *f* L at *e*.

To find the points *b c d* of the ſhadow of the inclined croſs D, from C through 4, draw a line at pleaſure; at 2 draw a line parallel to 1 4, cutting 4C at *a*, the ſeat of 2, from the point 2 make a line parallel to R, and from the ſeat *a* a line parallel to HL, cutting it at *b*; the point *c* is found by drawing 1*c* parallel to 2*b* or R, and 4*c* parallel to *a b*, the point *d* is found the ſame.

EXAMPLE XXXVII.

PLATE XXV. Fig. 2.

In this Example the Light comes from behind the Picture.

HL the horizontal line, O the center, on HL at H raiſe the perpendicular RH, R is the vaniſhing point of the rays of the ſun, and H is the vaniſhing point of the ſhadow on the ground; to find the points A, B, C, ſhadow of the ſolid figure D, draw a line for R, through the point 1, at pleaſure, and through *a* (the ſeat of *t*) and H draw a line interſecting R1 at A, the point B and C are found the ſame. Obſerve as O is the vaniſhing point of the ſide, 2 3, it is alſo of the ſhadow BC, the ſhadow of the croſs is found in the ſame manner; and the ſhadow of it over the block S is found, by continuing that

of

of the body of the croſs on the ground, till it interſects the ſide *r* of the block, and from that interſection, draw a line parallel to the upright of the croſs, till it comes to the ſurface at *t*; and from that point and H, draw the ſhadow over it, (as it runs parallel with that on the ground) every point is found as in Fig. D. The ſhadow of the cylinder is found by ſetting off as many points on it at pleaſure as *a*, *b*, *c*, find their ſeats on the ground as 5, 6, &c. then proceed the ſame as with the ſolid figure D, 5 is the ſeat of *c*, and 1 is the point of ſhadow, &c. Note, when the ſun is ſuppoſed behind the picture, the vaniſhing point of light is above the horizon.

EXAMPLE XXXVIII.

PLATE XXV. Fig. 3.

In this Scheme the Light from the Sun is before the Picture.

HV the horizontal line, C the center, V the vaniſhing point of the ſhadow on the ground plane, L the vaniſhing point of light coming before the picture; the point 3 the ſhadow on the ground of point *b* of the ſtairs, is found by drawing a line through its ſeat *a* to V, and another from *b* to point L, interſecting *a*V at 3 the point ſought. The points 1, 2, are found the ſame, *b*, *g* vaniſhes at C, as does the ſhadow 3 4. The ſhadow on the ſtep 3 *b c* is, by drawing 3 V, the ſhadow of the ſtick A on the ſtairs is projected by drawing AV, till it meets the bottom of the ſtairs; make the ſhadow *e* parallel to the ſtick where it meets the top, continue it towards V, the dotted lines from the head of the ſtick to point L, terminates the ſhadow of it on the flat top of the ſteps. The dotted lines of Fig. K, ſufficiently explain it, the ſide *e d* is parallel to the horizon, ſo is its ſhadow.

N. B. When the light is before the picture, the vaniſhing point of light is below the horizontal line.

EXAMPLE XXXIX.

PLATE XXVI. Fig. 1.

In this Example the Light is ſuppoſed before the Picture.

V is the vaniſhing point of ſhadow, W the vaniſhing point of rays of light, at any point on the quarter round where you want to project a ſhadow, make the ſection as by Example XXV. The point 4, ſhadow of *b*, is found by drawing *b*V cutting the quarter round at 3, there make a ſection, draw *b*W interſecting that ſection at point 4, point 2, ſhadow of *a* is found the ſame.

EXAMPLE XL.

PLATE XXVI. Fig. 2.

X, S is the horizontal line, X the center, S the vaniſhing point of the ſhadow on the ground, R that of light, come before the picture, CAB is a ſquare board placed on the top of a cylinder, *a b c* the ſeat of A, B, C, on the ground, and 1, 2, 3, 4 its ſhadow. The point J, the ſhadow of A on the cylinder is found by drawing A, S, cutting the top of the circumference, and from that point let fall a perpendicular, draw AR, interſecting it at J, the point ſought; ſet on as many points on ABC, as may ſerve to compleat the ſhadow, and proceed with every one in the ſame manner. E is the ſhadow of *e*, and *d* its ſeat on the ground, draw *d*S interſecting the bottom of the cylinder at D, there raiſe the perpendicular DF at pleaſure, draw *e*R cutting it at E, the line *d*DE is the ſhadow of the rail *d e*, and the points G, F, &c. are found as point E. The ſcheme is ſufficient to explain the reſt, the lines projecting the ſhadows being on the plate.

EXAMPLE XLI.

PLATE XXVI. Fig. 3.

In this ſcheme C is the center of the picture, H the vaniſhing point of ſhadow on the ground, and L the vaniſhing point of rays of light. To find

the

the points *q*, S, *y* ſhadow of K on the cylinder, from any point 2 on the circumference of the baſe of the cylinder (which is parallel to the picture) draw 2C, and from its ſeat *g* draw *g*C; then from the ſeat of K draw lines to H cutting *g*C in *f* and *u*, make *fy*, *uy* perpendicular to the ground line, cutting 2C at *q*, and *y* the points ſought; to find S take any point *r*, and proceed as with point Q; any points *n*, *m*, of the ſhadow of the circumference of the inward cylinder on its ſurface, are found thus; draw CL, and *at*, *bl*, parallel to it, cutting that circumference in *a t*, *b l*, then drawing *a*S, and *t*C, meeting in *n*, and *b S l* C meeting in *m*, *n*, *m*, are the points ſought, *h* being the ſeat of the point *s* on the outward circumference, draw 1L, *h*H, and their interſection is the ſhadow of point 1. Fig. 4. is a hollow cylinder cut open, every thing elſe ſtanding as in the foregoing example, *ar*, *bs*, *ct*, are parallel to CL, draw *r*C, *s*C, *t*C, and *a*L, *b*L, *c*L, interſecting each other at *e*, *d*, *f*, ſhadow of *a b c*, 5 is the ſeat of point 3, by drawing 3L and 5H interſecting it at 2, give 2 for the ſhadow of point 3 on the ground.

EXAMPLE XLII.

PLATE XIX. Fig. 1.

In this ſcheme VH is the *horizontal* line, V the *center*; make RS perpendicular to VH, S the vaniſhing point of ſhadow on the ground, R the vaniſhing point of rays of light. Then in the circumference of the arch ONB (which is parallel to the picture and vaniſhing at V) taking any point O, and finding its ſeat A on the ground, draw OR, AS, interſecting at *o* the point ſought; the point *n* on the wall E, K, B, C, ſhadow of N, is found by drawing through P (the ſeat of N on the ground) PS cutting EC, (the ſeat of the wall) in *p*, draw NR, and *p n* parallel to DL or EK, interſecting NR at *n*. In the ſame manner are got all the points that form the ſhadow of the arch; *look for point* R *at the bottom of the plate.* The points *d c f* in Fig 2. are found in the ſame manner as point *n* in the foregoing example; A is the

original point, B its ſeat on the ground, W the vaniſhing point of ſhadows on the ground, and A on the ground line; BD the vaniſhing point of rays of light, *c* the interſection, and *d* the ſhadow of A.

EXAMPLE XLIII.

PLATE XXVII. Fig. 3.

In this ſcheme the light is from the candle L, and L is the point of light, K its ſeat on the ground, and C the center of the picture; from K draw a line parallel to the ground line, cutting the ſeat of the ſide of the room TPX at T, draw TP parallel to KL, and LP parallel to KT, cutting TP at P, ſeat of light on that ſide the room, produce TP to X; make LV parallel to PX and XV, parallel to LP; point V is the ſeat of light on the cieling; W is the ſeat of light on the farther end of the room, and is found by drawing CK, where it cuts the interſection of that ſide of the room and the floor, raiſe a perpendicular, and draw CL, interſecting it at W. Having the different ſeats of light, proceed in projecting the ſhadows which are found in the ſame manner as from the ſun. Point *b* ſhadow of *d*, is found by drawing through K, and the leg of the table, a line at pleaſure, interſecting it by L*b* at *b*, and every point of ſhadow on the ground of the table is found the ſame way; point *p* ſhadow of *y*, is found by drawing L*p* at pleaſure, and through the ſeat of *y* and P, draw a line cutting L*p* at *p*, in this manner you may finiſh the ſhadow on the wall, of the circle I *y*. To find point 8 on the ſide of the room, thro' K and 4, draw a line cutting the ſeat of that ſide of the room at *t*, and draw a line through L and the top of the chair at pleaſure, then draw 7 8 perpendicular to the ground line interſecting at 8. The points 1, *h*, *k*, on the cieling, ſhadow of the croſs *e*, *g*, *f*, are found by drawing through the ſeat of *e*, *g*, *f*, and W, lines cutting the interſection of the cieling at *n*, (the croſs being perpendicular to the picture, its vaniſhing point is C) then through *n* draw C*nh*, and through every interſection for the points *i*, *k*. The ſhadow of the ball on

the

the cieling is better underſtood by Fig. 3. *l* is the point of light, and S its ſeat, C the center of the ball, draw C*l*, divide it in two equal parts in *k*; ſet on the foot of the compaſſes in *k*, with the opening *k*C, cut the circumference of the ball in E and F, draw the line E*x*F. On the diameter 3C4, make the circle 1 2 3 4, parallel to the ground plane, and make another paſſing thro' the points 1, 2, and the line C*k*; thro' the points *x* and V, draw a line cutting the circle 2 6 5 1; at 5 and 6, and make the circle E6 5F. That upper part of the ball is as much as can be enlightened by a light placed at *l*, or as much as can be ſeen by a ſpectator at that diſtance. Find the points *e* 6 5 *f*, ſeats of E, 6, F, 5; then from S the ſeat of light, and through *e* 5 6 *f*, &c. draw lines parallel to the horizon, interſecting them by lines thro' *l* and the points E6, F5, at *n*, *r*, *m*, *s*, join thoſe points for the ſhadow of the ball on the ground.

EXAMPLE XIV.

Fig. 2. In this ſcheme R is a ray of light from the ſun parallel to the picture, V the center of the picture, 1 6 4 5 a ſection parallel to the ground plane, 1 2 3 4 another perpendicular to the line R (as the rays from the ſun are to be conſidered as parallel, one half of the ball is enlightened.) The ſhadow of *p n l t* on the ground is found by drawing thro' every point of the circle 1 2 3 4 lines parallel to R1, as V3, P9, and lines parallel to the horizon, paſſing thro' their ſeats, interſecting them at *p n l t*, the little circle on the ground is the ſeat of the great one 1 2 3 4, and is marked by the ſame figures.

CHAP.

CHAP. VI.

Of finding the Reflection of any Figure on poliſhed Planes or Standing Water.

EXAMPLE XLIV.

PLATE XLV.

It is well known, that the reflection of figures on a poliſhed plane, or on the ſurface of ſtanding water, appear to be juſt as much on one ſide of the plane, as the real objects are on the other ſide: ſo that to find the reflected repreſentation of any point, you muſt draw a perpendicular to the reflecting plane from the real point, and in it take a point at the ſame diſtance on the contrary ſide of the plane. For inſtance, to find the reflection of the perpendicular AB (Plate XXXVII.) let AC be the ſurface of the reflecting plane, and A the ſeat of B, continue the perpendicular AB downwards, till A*b* is equal to AB. The angle any original line makes with the reflecting plane, is the ſame as the reflection with that plane.

As in F (Fig. 2.) DC the ſurface of the water, the angle CDF is equal to the angle CD*f* of the reflection. Vaniſhing points of any real objects are the vaniſhing points of their reflections, as O, Fig. 3. is the vaniſhing point of the ſquare board, and its reflections: the conſtruction of that figure is ſufficiently clear by the ſcheme only.

EXAMPLE XLV.

PLATE XXVI. Fig. 3.

In this example the ſurface of the water is ſuppoſed even with the ground, or the line *b g*, and the circles in the reflection are the ſame ſize as the real ones, and the ſame diſtance from the line *b g* downwards, C is the vaniſhing point, and every point of ſhadow in the reflection may be found

in

in the ſame manner, as *nm* in the real figure, making *l*H equal to HL, and uſing the point *l* inſtead of L.

CHAP. VII.

Of the Perſpective of Landſcapes.

EXAMPLE XLVI.

PLATE XXVIII. Fig. 1.

In this example is repreſented a landſcape that has a deſcent marked G*ab*1, a riſing ground *e d* K *n*, and GC1 a flat country, HL the horizontal line, C the center, and L the diſtance, BF the vaniſhing line of a plane inclined below the horizon, LB its inclination, B the center, and F the diſtance on it, AS the vaniſhing line of a plane inclined above the horizon, LA its inclination, A the center, S the point of diſtance on it; draw GB, IB, and GF, cutting IB in *b*, make *b a* parallel to GI. For the ſquare piece of water, draw *a*C, *b*C; then *a*L, or *b*H, cutting *a*C at *c*, make *c d* parallel to *a b*. For the aſcent, draw *c*A, *d*A, interſecting *c*A by *d*S, at K, make K*n* parallel to *c d*. GK*n*I being perpendicular to the picture, vaniſhes at C, and D is the vaniſhing point of the houſe E. To find point *k* in the water (*a o* being the ſurface) draw the perpendicular K*k* at pleaſure, and *a* C cutting it at *r* (*r* is the ſeat of K on the water) make *r k* equal to *r* K, the points are found the ſame, *ck* is the reflection of *c*K, and *k s* of K *n*.

The ſhadows which are ſuppoſed to be caſt by the ſun, and parallel to the picture, the line R the inclination of the rays of light, make *l i* parallel to it, and *b*1 parallel to the horizontal line, join I *i b*. For the ſhadow of I *l* on the ſlope, draw *i m* to the point C, cutting *c d* in *m*, join *n m*, then I *i m n* is the ſhadow of I *l n* on the down-hill ſurface of the water and riſing ground. The poſts T, U, and their ſhadows, vaniſh at B. Plate XXIX. will ſufficiently explain the reſt.

EXAMPLE XLVII.

PLATE XXVIII. Fig. 2.

In this example HL is the horizontal line, C the center (or the point of ſight removed in the picture) GD the ground-line, B a riſing ground, ND the ſeat of it, NB the perpendicular height at B, equal to DA on the ground-line, KNI being produced towards the horizontal line till it cuts it, that point is the diſtance; and if KN, GD, be continued till they meet, from their interſection to D will be the real diſtance between the point D and point N. The level ground at a great diſtance riſes to the horizontal line; the tops of the hills higher, but their ſeats are below it. The light which is from the ſun is parallel to the picture, and the line R the inclination of the rays of light, every ſhadow, and reflection in the water is found as in the foregoing example. Plate XXX. repreſents the ſame finiſhed.

EXAMPLE XLVIII.

PLATE XXXI.

In this landſcape are repreſented ſeveral ſtreets, whoſe vaniſhing points are H, C, L placed on the horizontal line HL, C is the center of the picture; the houſes ſtanding obliquely with the picture, have one ſide vaniſhing at point L, the other at H; the houſe F having one ſide perpendicular to the picture vaniſhes at C; the ſtreets are ſuppoſed to be at right angle with each other; O an octagon baſon, the perſpective of it is found by the points H, *d*, *e*, L on the horizontal line, and *l m* as diameter On VH, perpendicular to HL, are ſet the vaniſhing points Q S. The ſide of the roofs vaniſhes at S, the ſhed C at Q, and the roof *v* of the houſe *e* at V; the oppoſite ſide forming the angle vaniſhes at a point as far below the horizontal line as the other is above it. Make *k*C perpendicular to H L, continue one ſide of the pediment *g* of the houſe F, cutting *k*C at *k*, the vaniſhing point for that ſide. If lines are drawn from the top and bottom of the houſe *e* to the point L, any line perpendicular to HL between thoſe lines, as

1, 2, will be equal to the height of the houſe *e*. Thro' 1 and 2 draw lines to H, for the top and bottom of the houſes in that ſtreet, the line RL is perpendicular on HL, and *h* is the vaniſhing point of *z* the roof of the church B. The light is parallel to the picture, and to the line RH; ſo that the ſhadow *x* of the point *g* is found by drawing 3*x* through its ſeat parallel to HL, and *g x* parallel to HR cutting it in *x*. All the rays being parallel to RH, and H being the vaniſhing point of *t a* (the ſide of the houſe before the church) HR is the vaniſhing line of the plane made by the rays which paſs through the line *t a*, and LR being the vaniſhing line of the church B; the ſhadow *t u* is caſt on, R is the vaniſhing point of the common interſection of thoſe two planes, that is of the ſhadow *t u* of the line *t a*; therefore the line *t u* is drawn through the point R, and not parallel to HR as *g x* for the ſhadow 3*x*; through Q draw *u s* for the continuation of the ſhadow *t u*, of *t a* on the top of the ſhed, and *s w* perpendicular to HL finiſhes the ſhadow of *t a* and the ſide of the houſe.

EXAMPLE XLIX.

PLATE XXXII.

In this landſcape the houſes A, B, D, E have one ſide perpendicular to the picture, therefore vaniſhes at C, the center; the dotted lines drawn through the bottom of the houſes is their ſeat, (which is even with the ſurface of the water) make every reflection as far below that line as the real object is above it, as appears by the Fig. 1, 2, 3, in the water anſwering the ſame figures above it; point O is the vaniſhing point for the boards F and its reflection *f*; the light is ſuppoſed coming from behind the picture, O is the vaniſhing point of the ſhadow on the ground, but the vaniſhing point of the rays of light are out of the plate, to ſhorten the ſhadows. Plate XXXIII. repreſents the ſame finiſhed.

EXAMPLE L.

PLATE XXXIV.

In this plate are ſeveral repreſentations of balls or ſpheres; moſt of their ſections being parallel to the picture are ſimilar to their originals; and tho' the contour that ſhould form the balls round are not ſo, yet their appearances are round to every ſpectator. That repreſentations and their appearances may be different, is very evident, by what has been demonſtrated at the end of the theory (See Plate XXXIX. Fig. 1.) where CD is the repreſentation, and C*d* the appearance; for unleſs the axis of the ſphere paſs through the ſpectator's eye, and center of the picture, the repreſentation of it cannot be round, if according to the ſtrict rules of perſpective. Fig. 6. in this plate ſhews *d e* for the repreſentation of the line D*w*, and *c* of point C (the center of D*w*) on the picture HK, but *c* on the line *d e* is not in the middle. But *h k* the appearance of D*w*, and *i* of C to a ſpectator's eye, placed as in the ſcheme, *i* appears to be the middle of *h k*.

EXAMPLE LII.

PLATE XXXV.

Repreſents a landſcape with a riſing ground that aſcends towards the horizon, and anſwers to Plate XXX. That in Plate XXXVI. anſwers to Plate XXXIII.

EXAMPLE LIII.

PLATE XXXVII. Fig. 2.

Let AB be the horizontal line, C the center of the picture, and GD the ground line. Take J the middle of the line AB as diameter, and deſcribe a ſemicircle AOB, at C raiſe CO perpendicular to AB, cutting the circumference at O point of ſight: Let A be the vaniſhing point of H*g*. To determine

termine the point *g*, take AO for a radius, deſcribe the arch OU, and AU will be the diſtance; ſet any point G on the ground line, and draw GU cutting H*g* in *g* the point ſought: in this manner may be cut any line in the picture: if B is the vaniſhing point, F is the diſtance, BO being radius, and OF the arch; make HD equal to HG, draw HB and FD, cutting HB at *d*, then draw *d*A, *g*B interſecting at *h*, and H *g h d* is the perſpective of a ſquare, whoſe ſides are equal to HG or DH, and the angle a right one, as AOB.

EXAMPLE LIV.

PLATE XXXVII. Fig. 1.

In this example every thing ſtanding as in the laſt, on the ground line ſet any meaſure given, as HD for the front of a building, IL the center part of it, K the middle of the pediment, MG its depth, and MH the diſtance the corner of that building is in the picture. Draw HF, MU interſecting at *h*; draw *h*B, *h*A, and from the points I, K, L, D, draw lines to point F, cutting *h*B at *i*, *k*, *l*, *d*, and from the points M, N, G to U cutting *h*A at *ng*. To give the proper height of every parts of the building, produce *h*B to the ground line at P, make PQ perpendicular to GD, ſet on it the ſeveral dimenſions, and on *h*, *i*, *k*, *l*, *d*, *n g* raiſe lines perpendicular to the ground line, and through every diviſions on PQ, draw lines to B cutting the perpendiculars *h*, *i*, *k*, *l*, *d*, at *b*, *r*, *x*, and through the points *b*, R on *h*, draw lines to A cutting the perpendicular *n* at T, and *g* at V, join *b*TV for the roof. To find its vaniſhing point, make EA perpendicular to AB; produce *b*T, cutting EA in E for the vaniſhing point of the ſide of the roof *b* T *x u*. If EA be continued below the horizontal line, and TV produced, their interſection will be the vaniſhing point for TV. To finiſh the pediment *w f u*, draw *k*A, *n*B at their interſection *s*, raiſe a perpendicular to GD, draw TB, cutting it at *y*, then draw A *i o*, A *r w*, make *o w* parallel to *i r*, draw *o*B, *w*B,

draw

draw a line through A and *l*, cutting *o*B at *z*; make *z u*, parallel to *w o*, cutting *w*B at *u*, through KF draw a line cutting *o z* at *e*, at that point raiſe a perpendicular to GD, and through *y* and A draw *y f*, cutting *e f* at *f*; then join *w f u*, draw *a t* B (the top of the window) till it cuts *i r* at *t*, then draw A *t c* cutting *o w* at *c*, draw *c*B for the top of the window on that part of the building, proceed in the ſame manner for the under part of them; for the windows on the ſide F of the houſe, it is only uſing the point A inſtead of point B. Theſe examples are ſufficient for the young practitioner to draw landſcapes, views, &c. &c. in perſpective, if he has already acquired ſome ſkill in drawing, as by theſe rules he will be enabled to adjuſt and correct his deſigns, and be capable of performing them to the greateſt nicety and exactneſs.

F I N I S.

20 MR 51

E R R A T A.

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16	27	Fig. 2.	Fig. 4.
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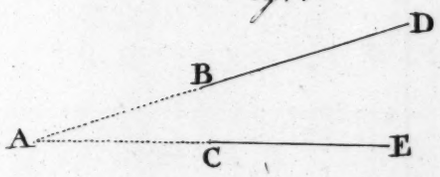
52	7	parallel to it	parallel to the picture
54	10	on	C2, 1D.
55	6	45 *do*	45 deg.
56	25	for R	from R.
60	3	dele the ſemicolon after the word *line*.	
64	25	roofs	roof *ſ*.

Fig: 1



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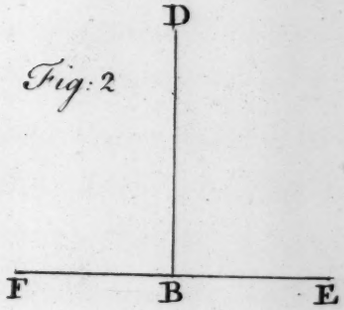

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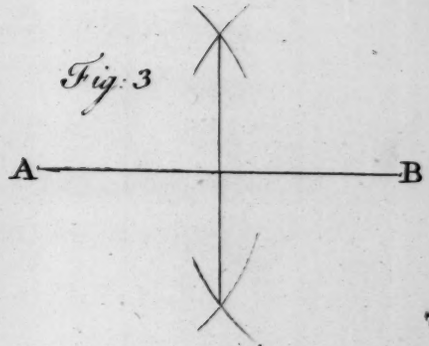


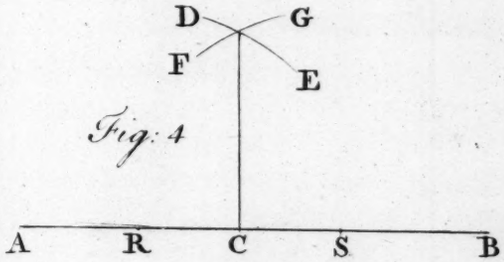
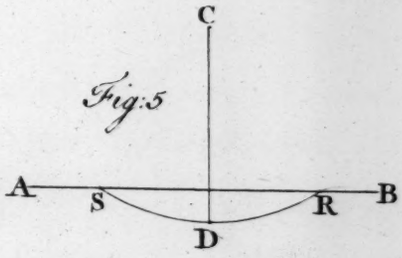


Fig: 4

Fig: 5



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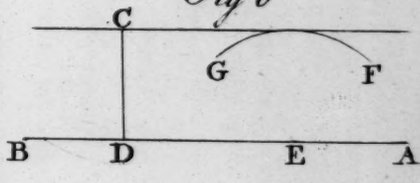

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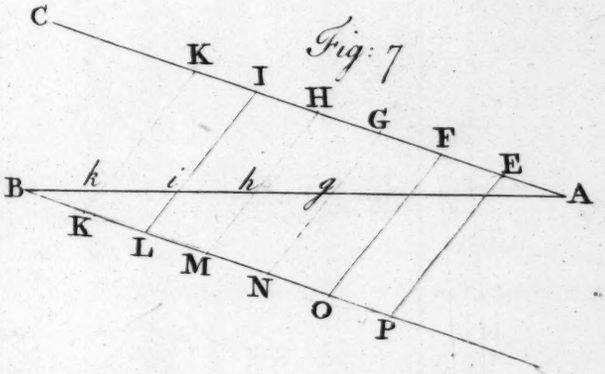

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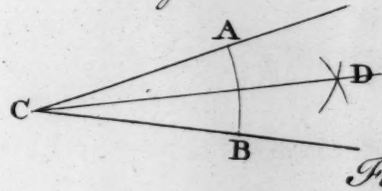

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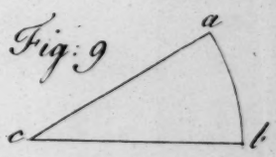

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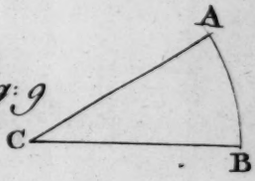

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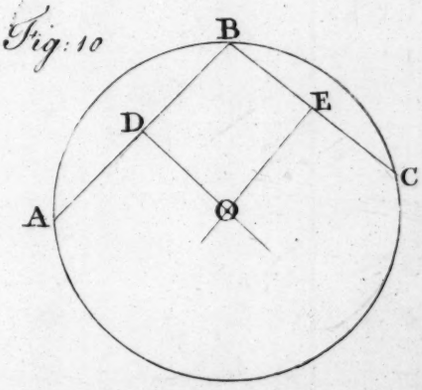

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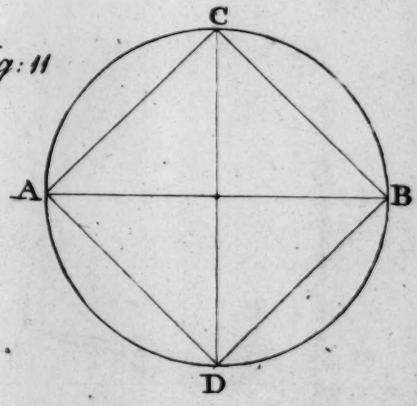
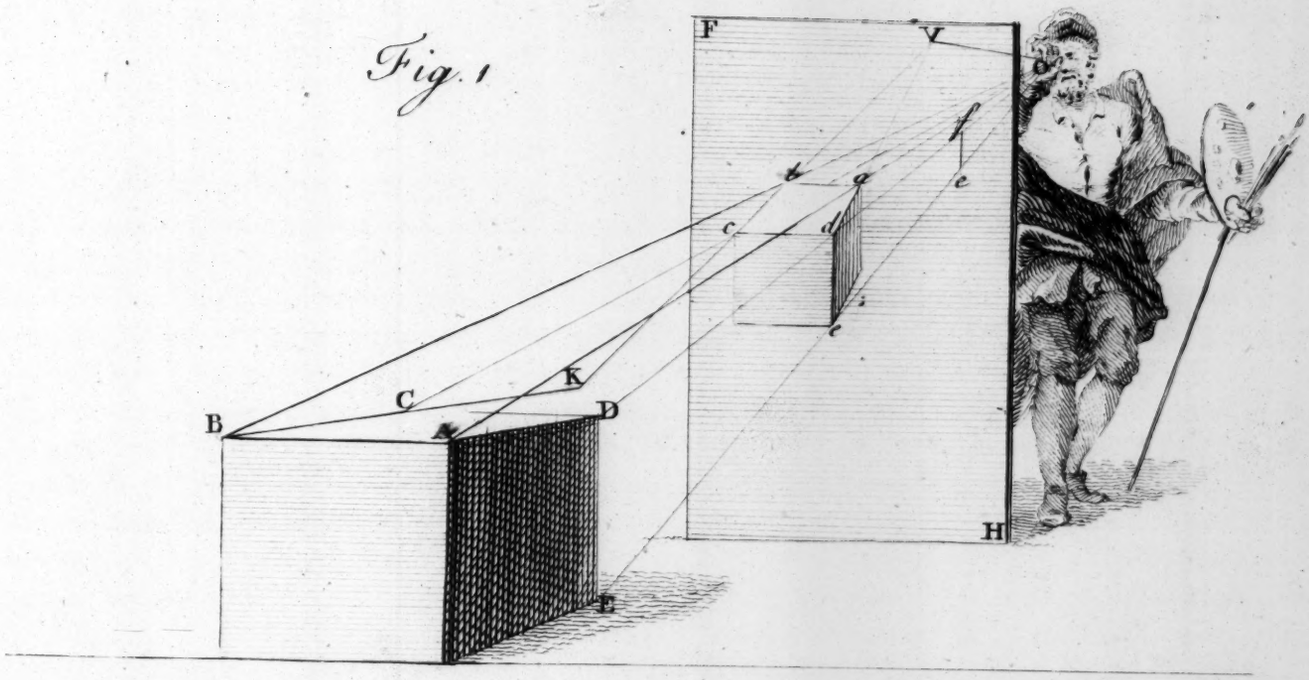
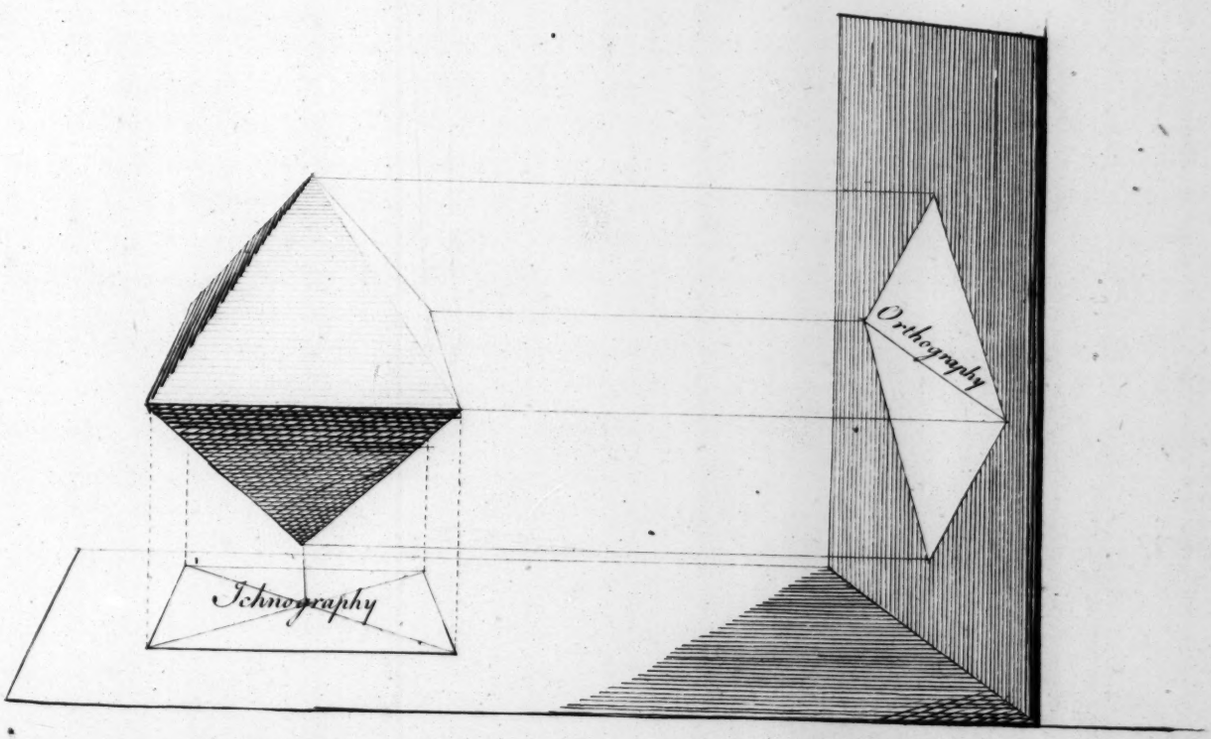
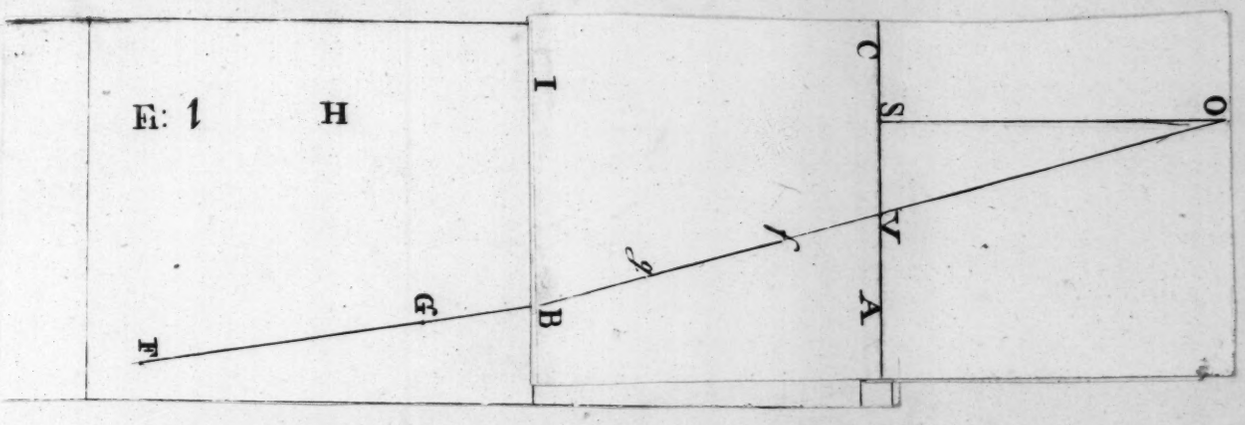
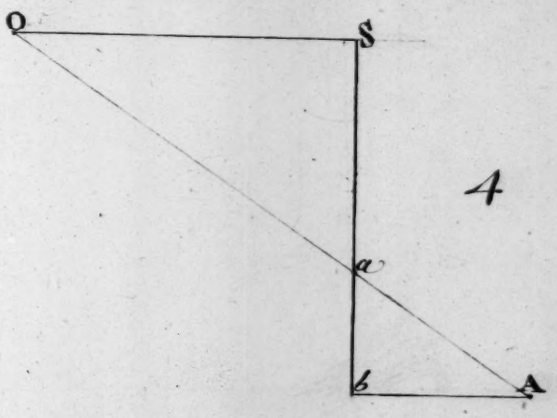


Fig. 1









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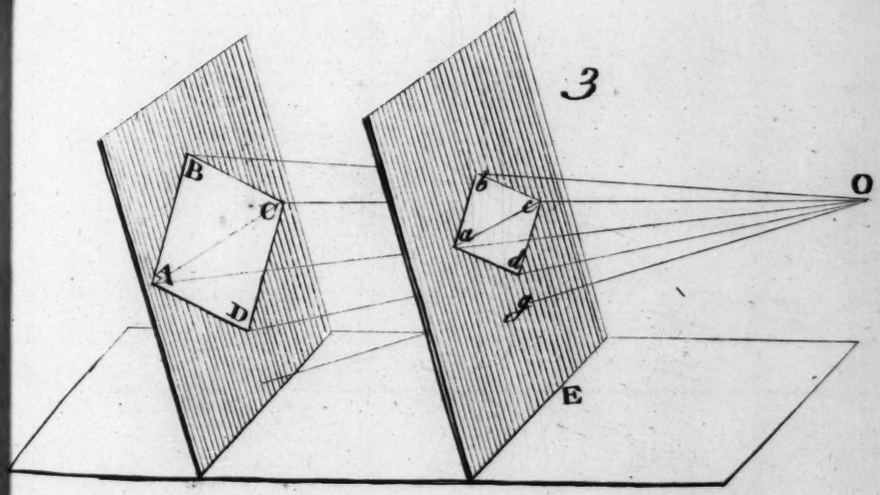

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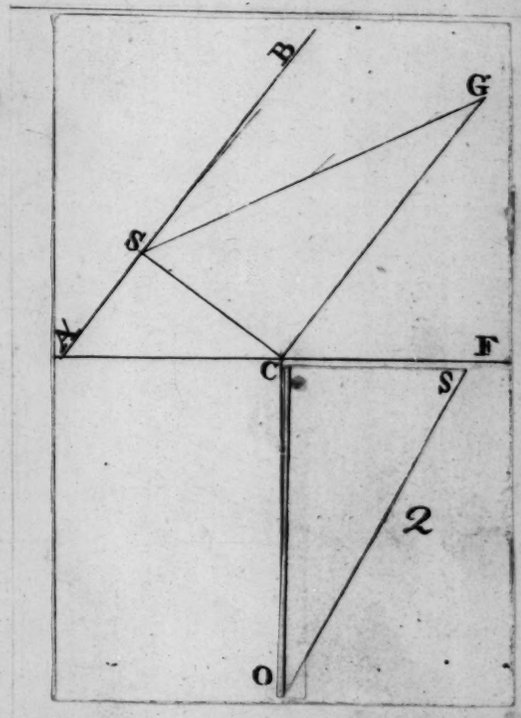

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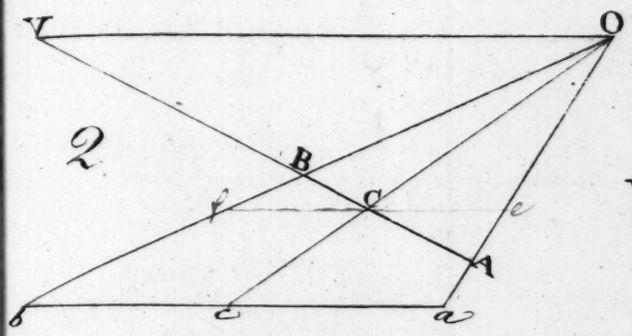
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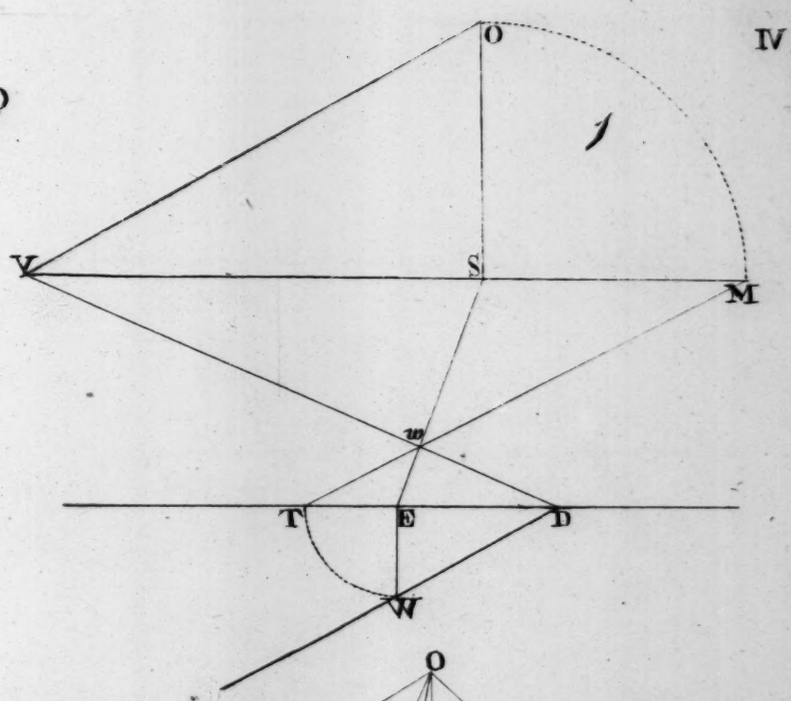
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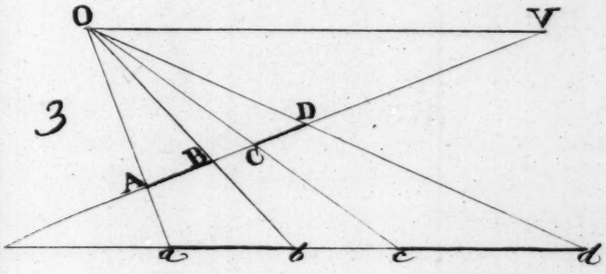
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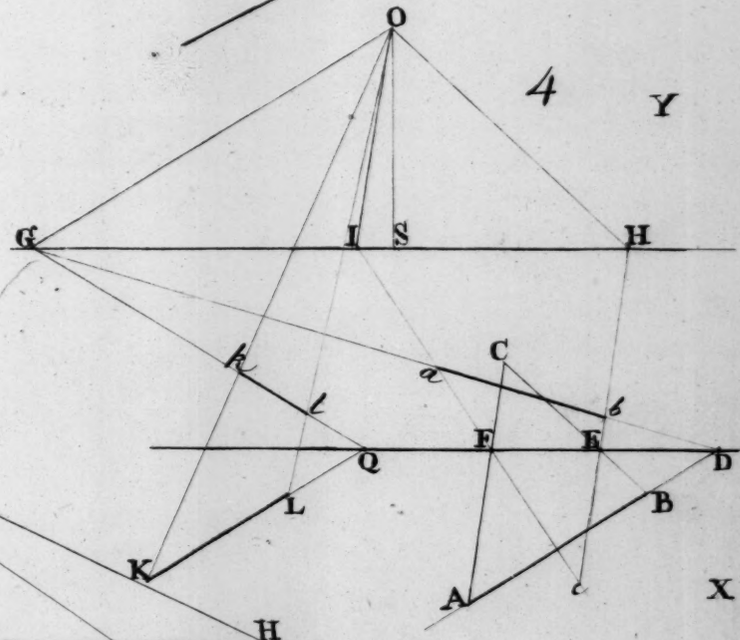
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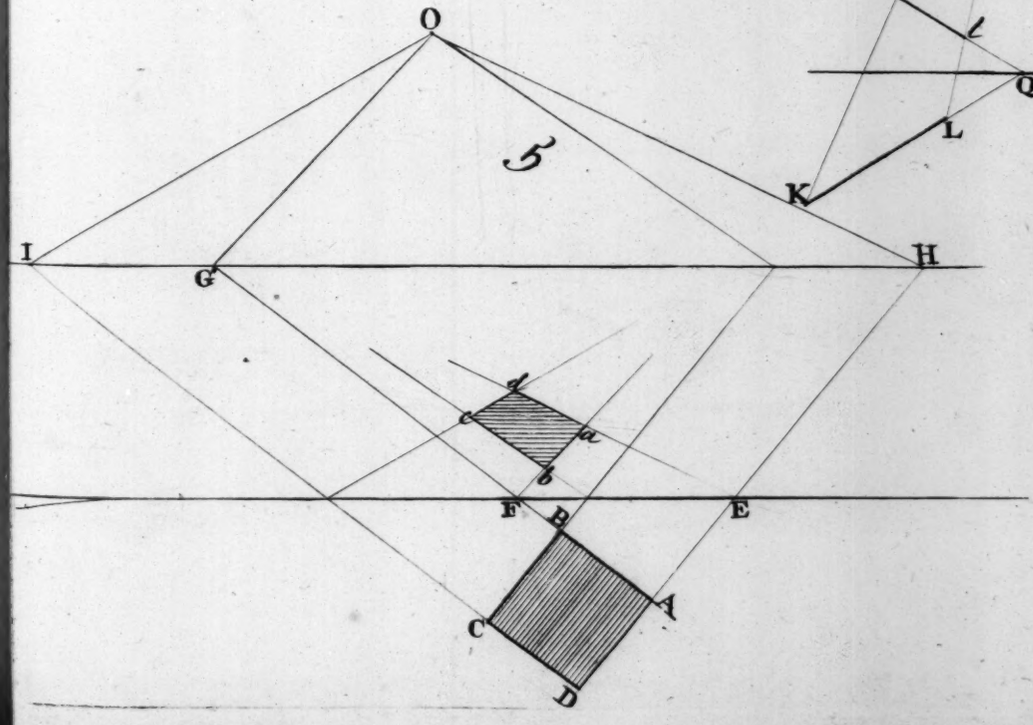
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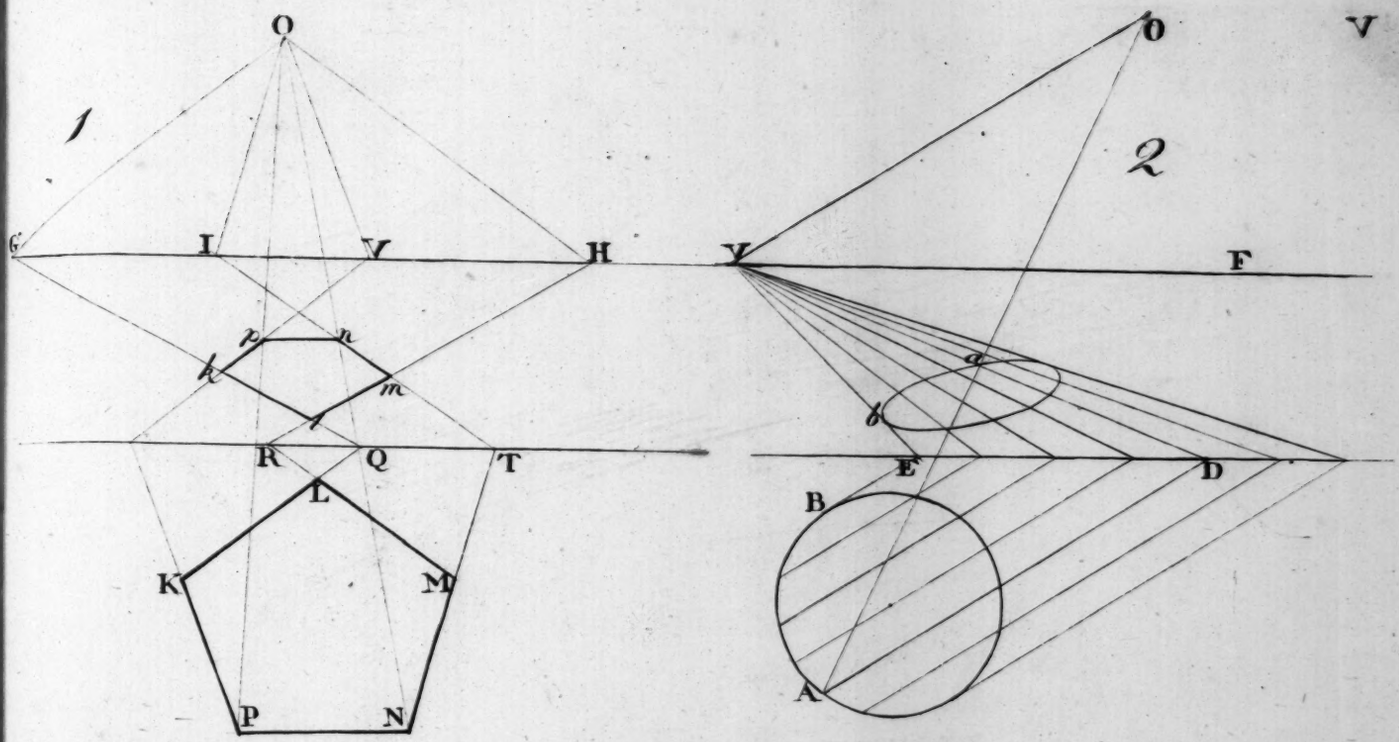
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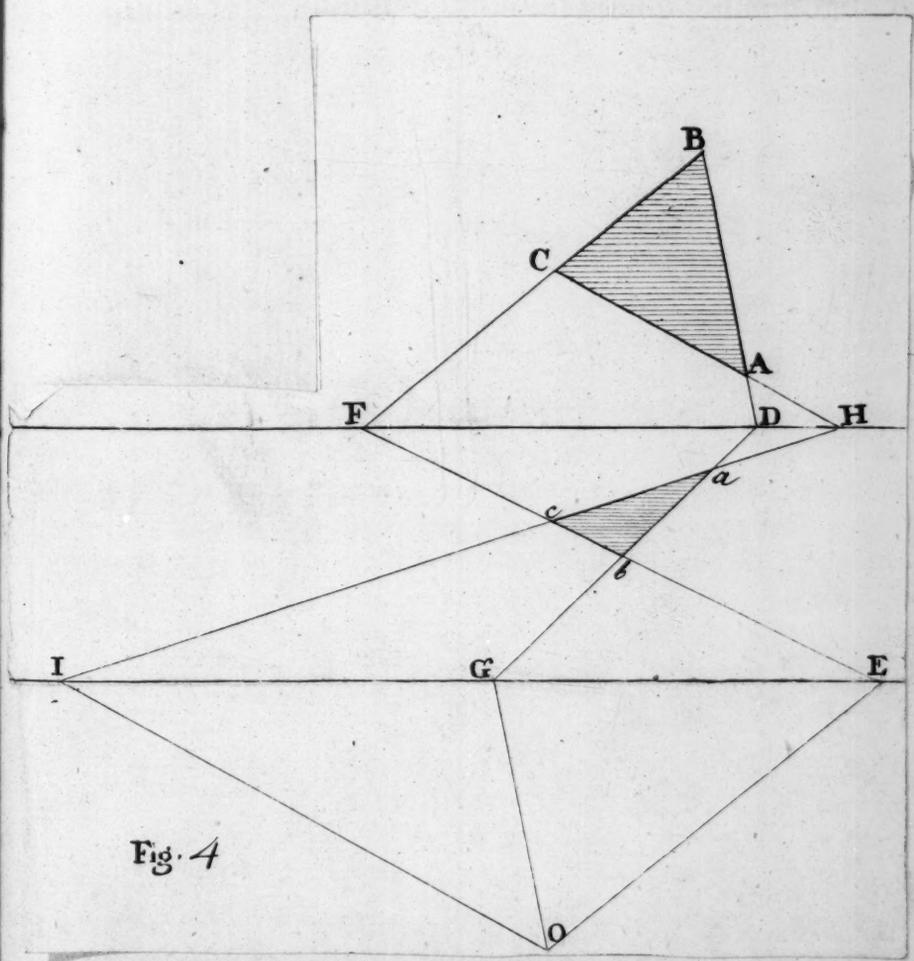
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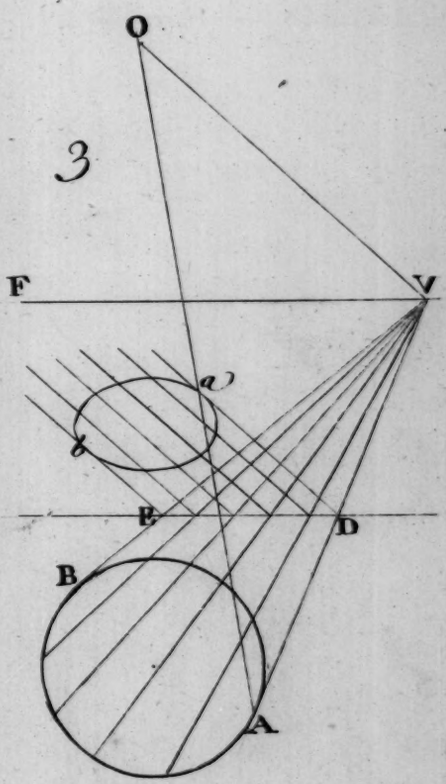
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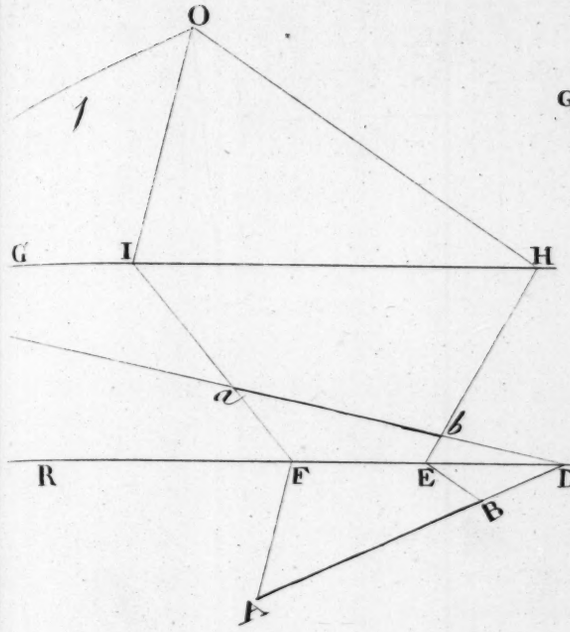
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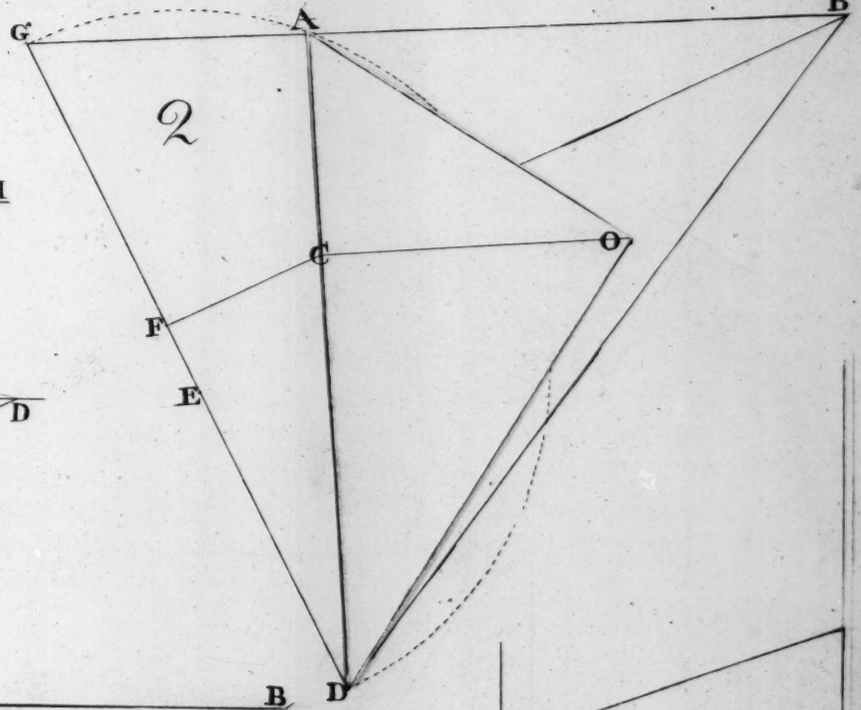
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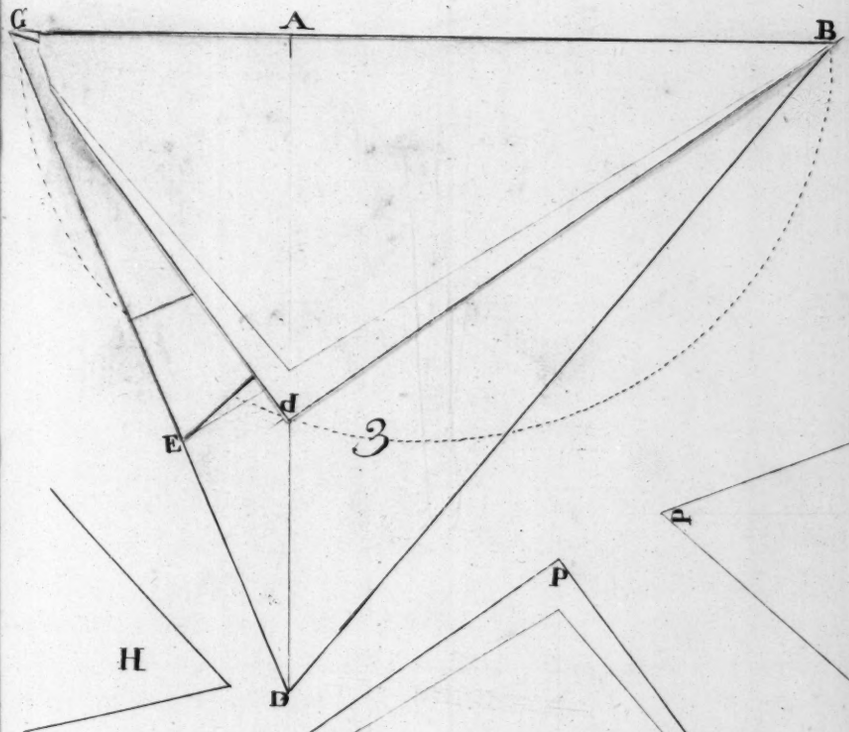
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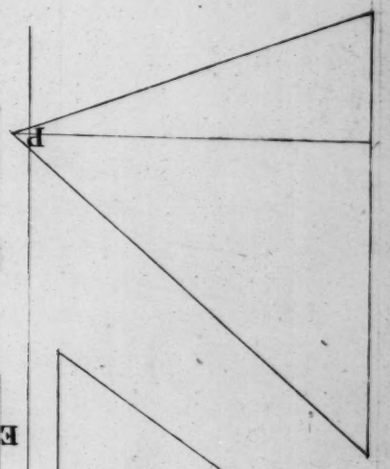
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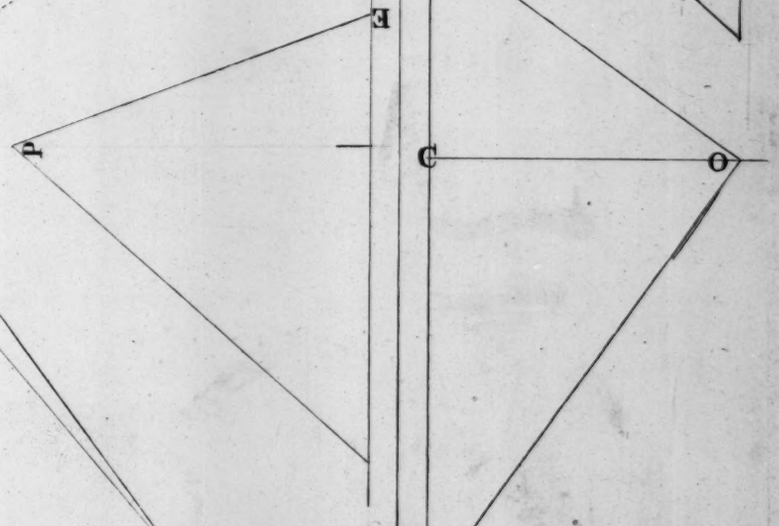
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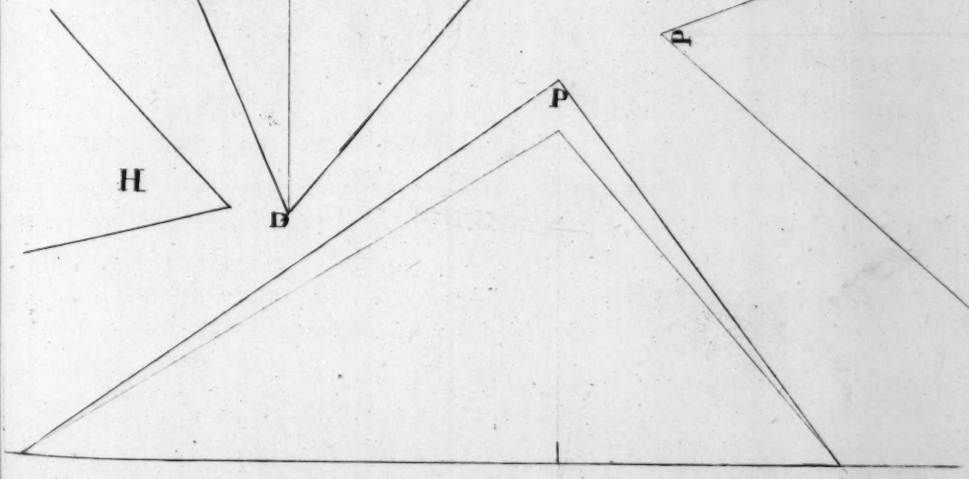
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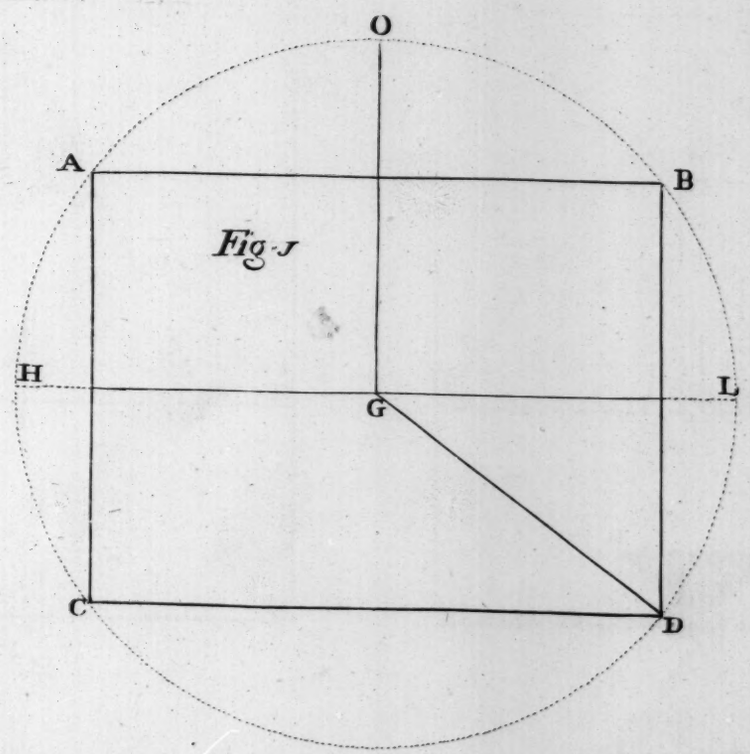

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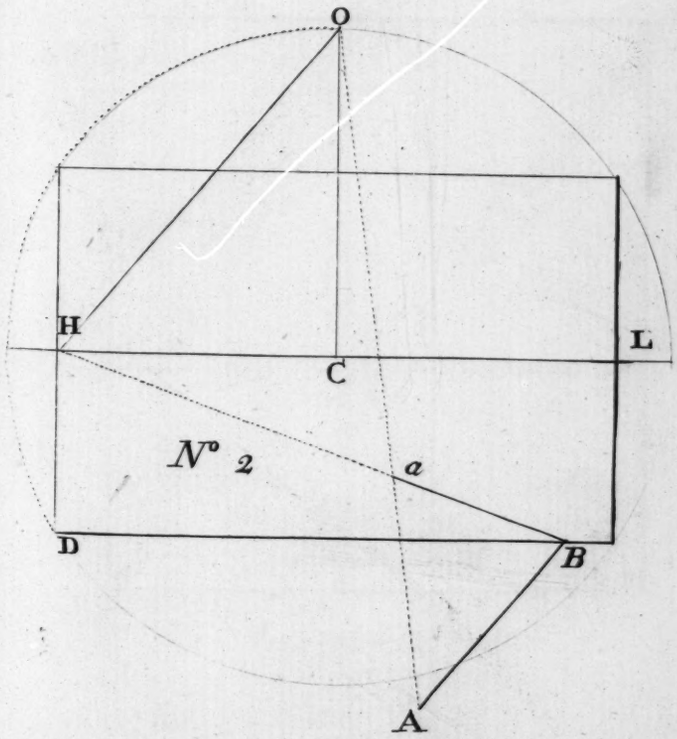

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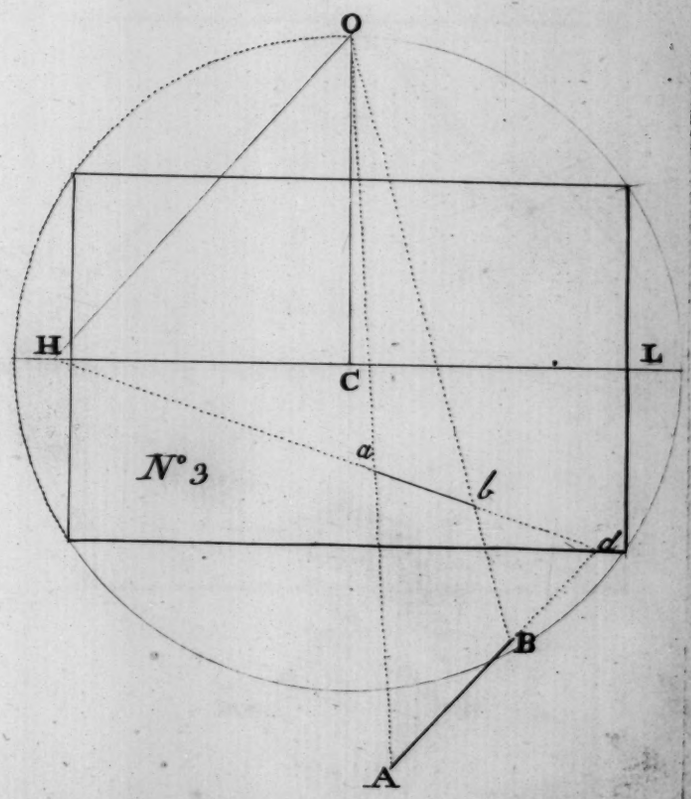

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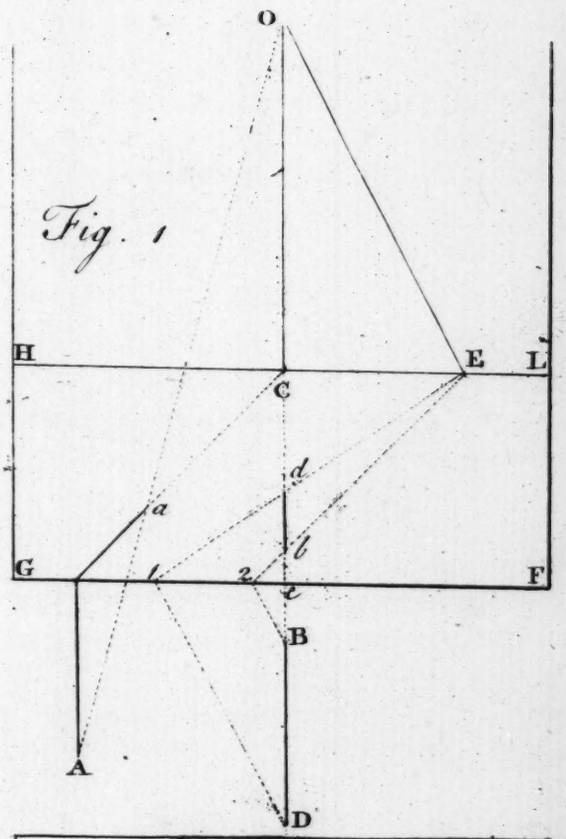
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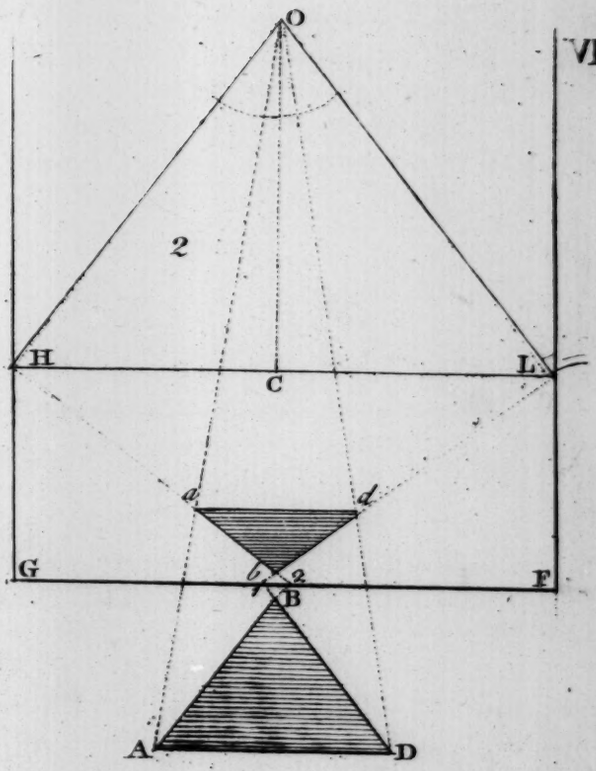
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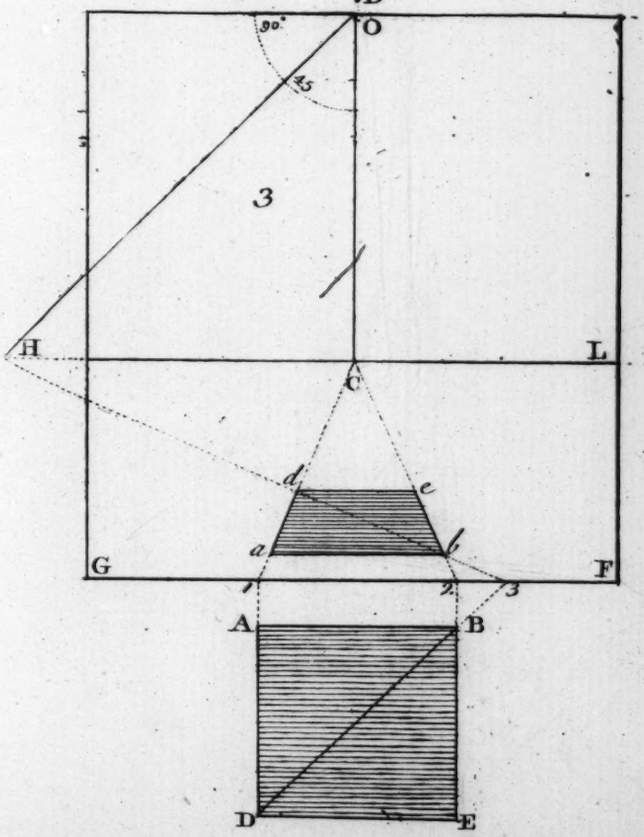
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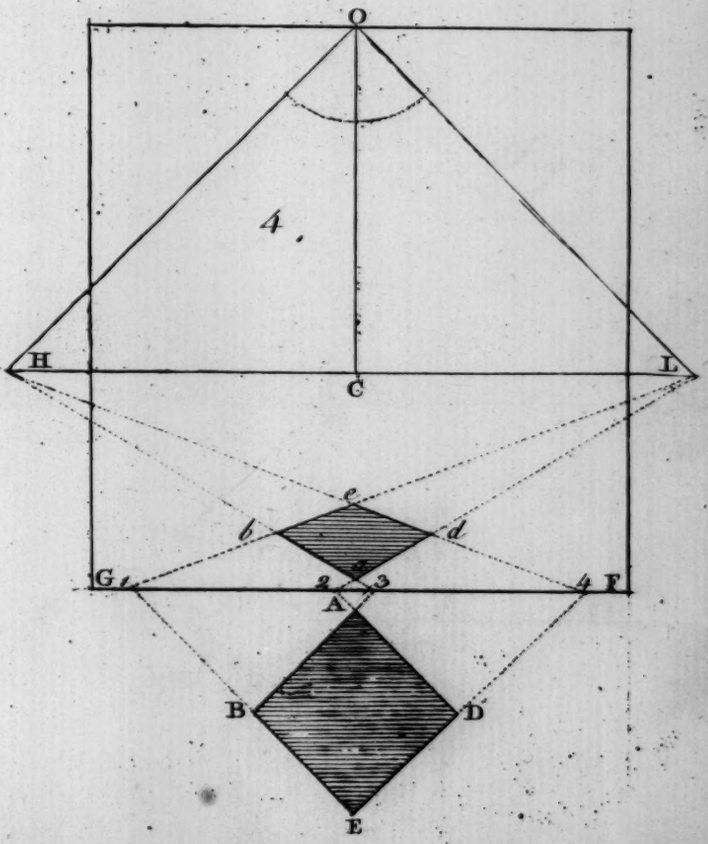
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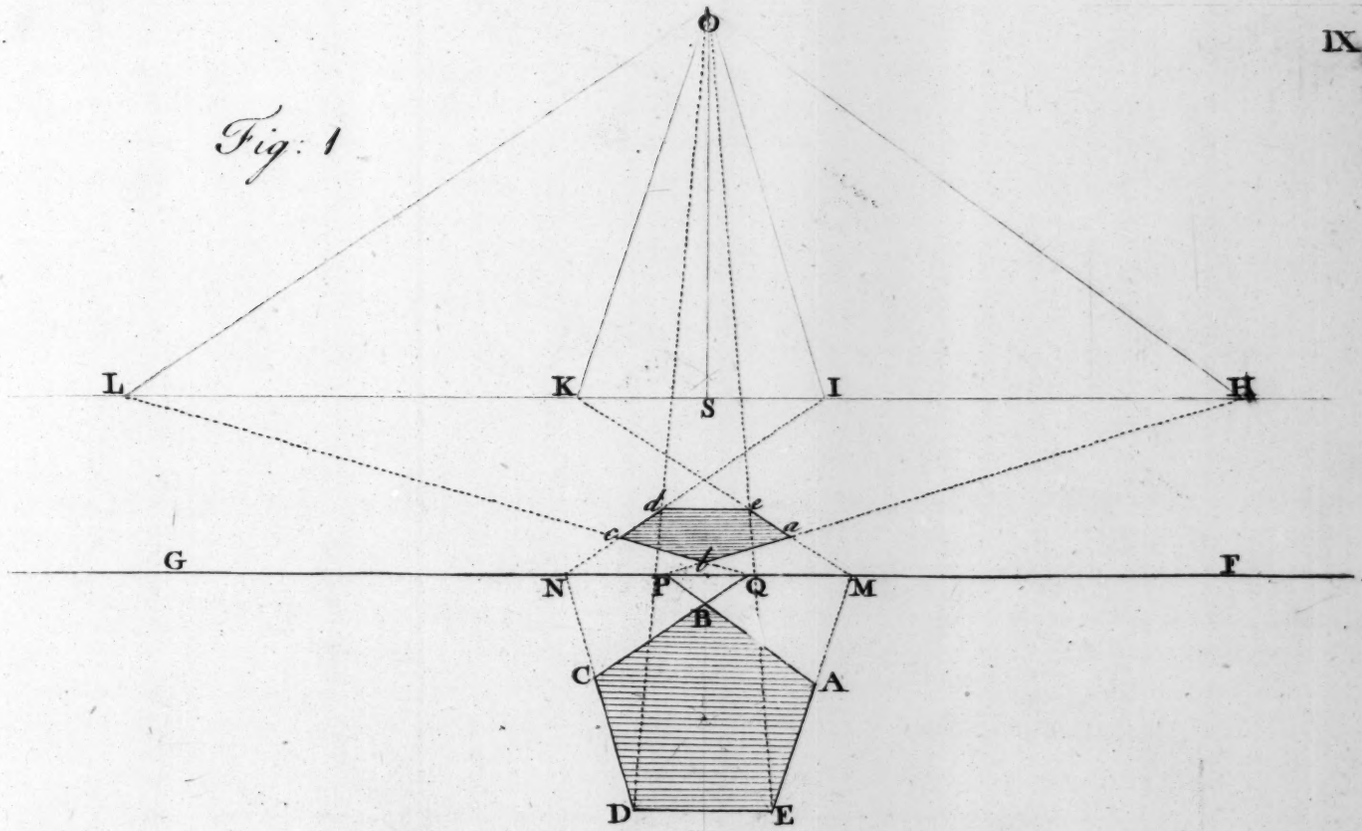

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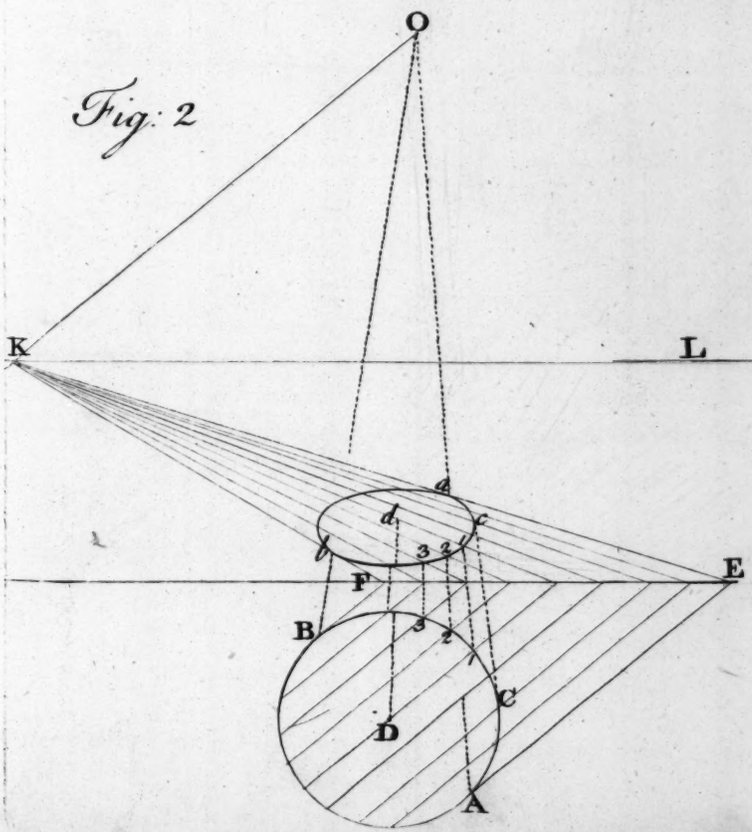

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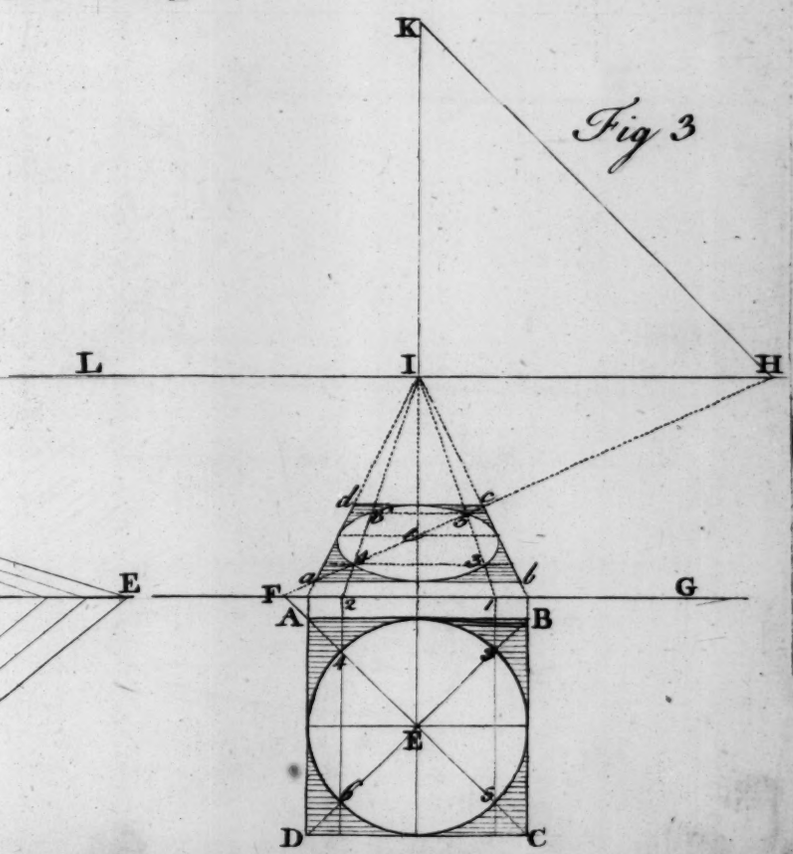

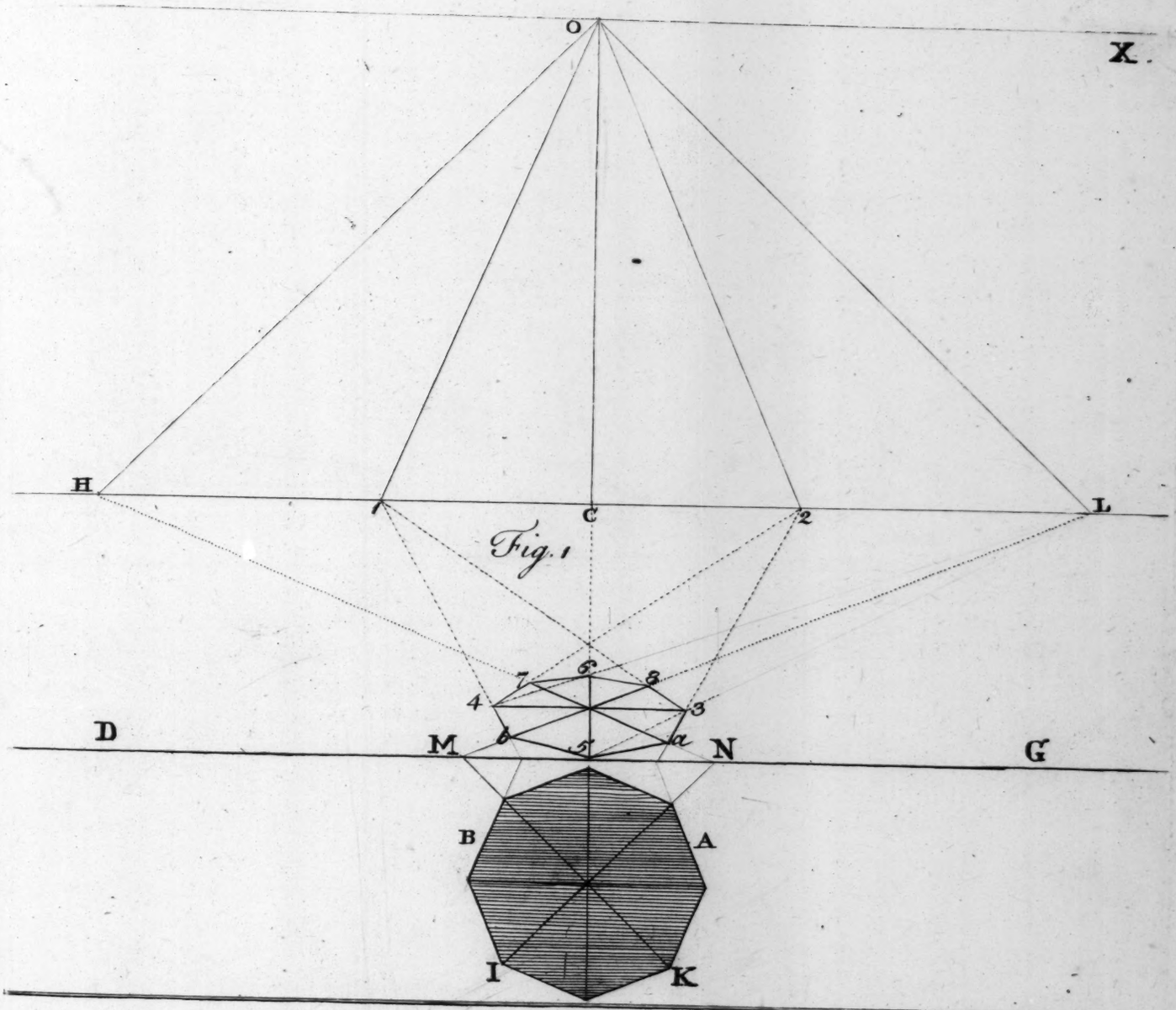
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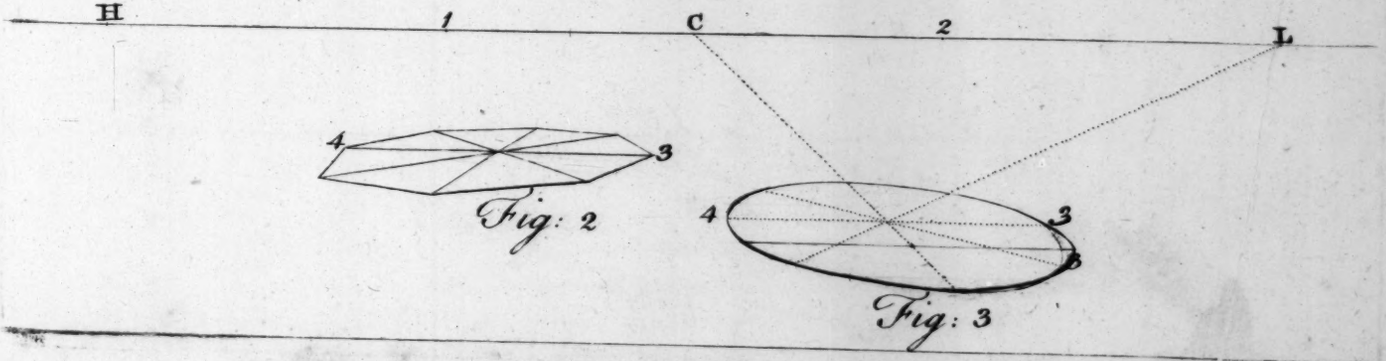
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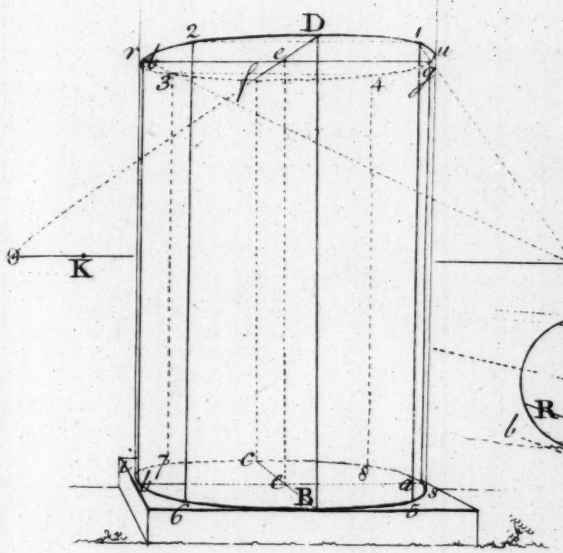
Fig. 1



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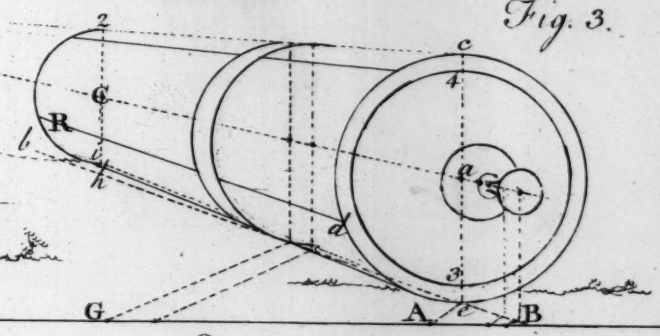

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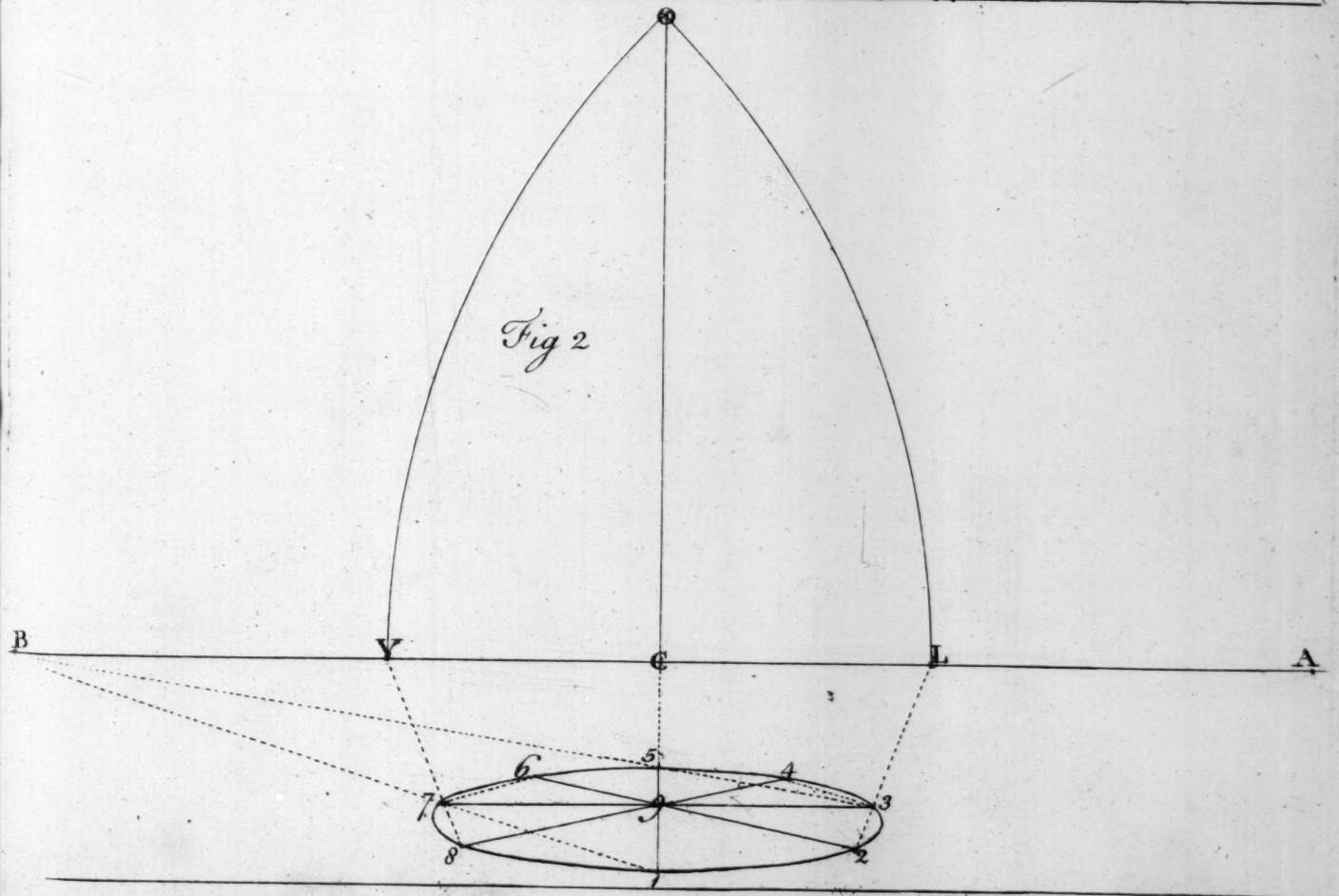

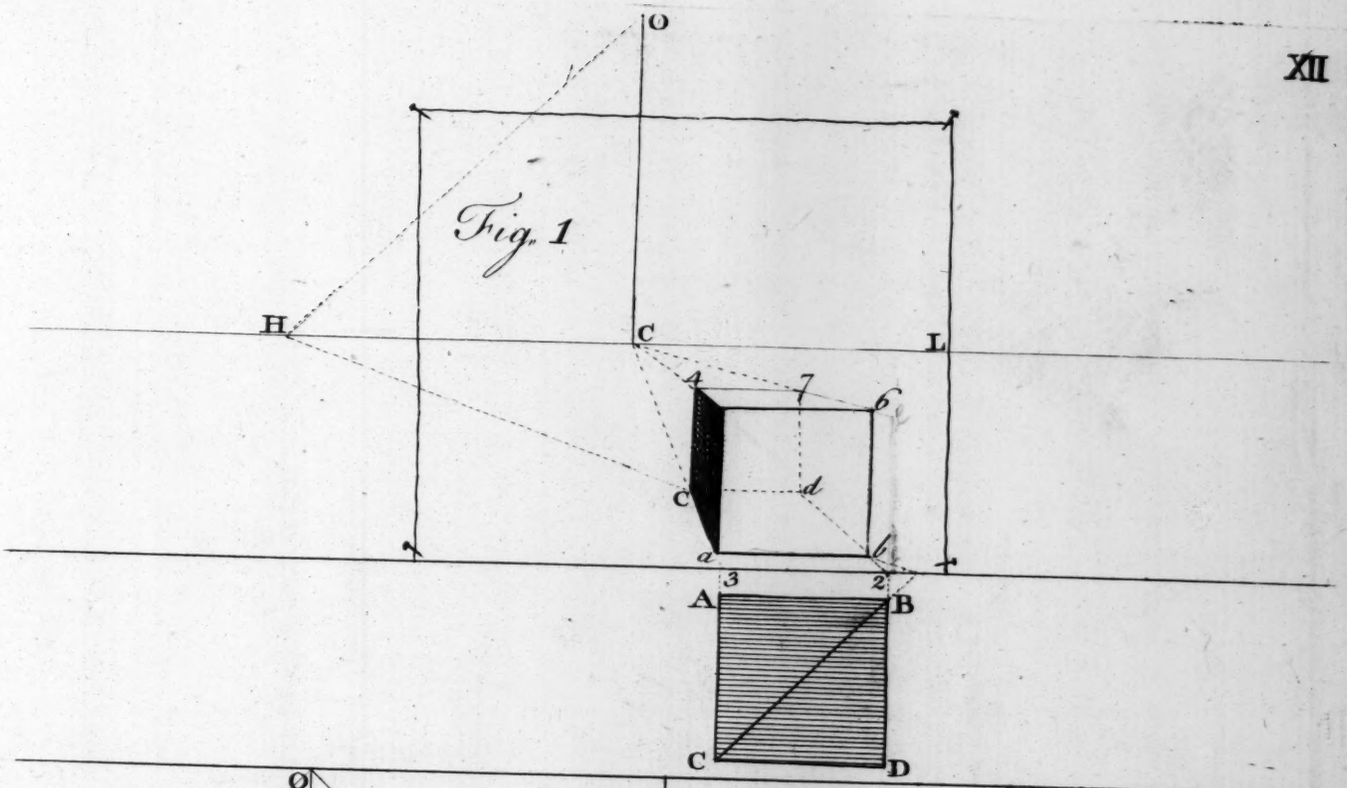
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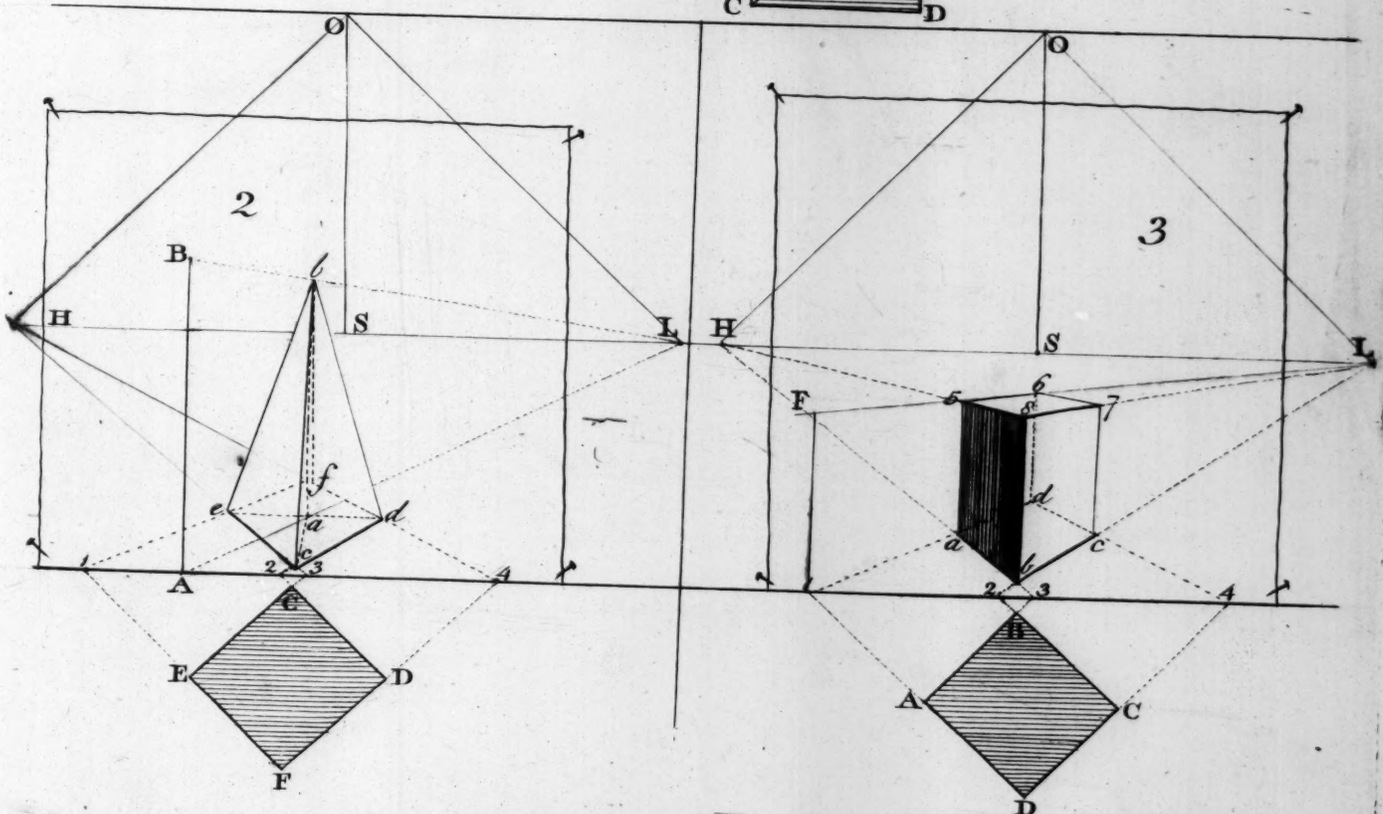
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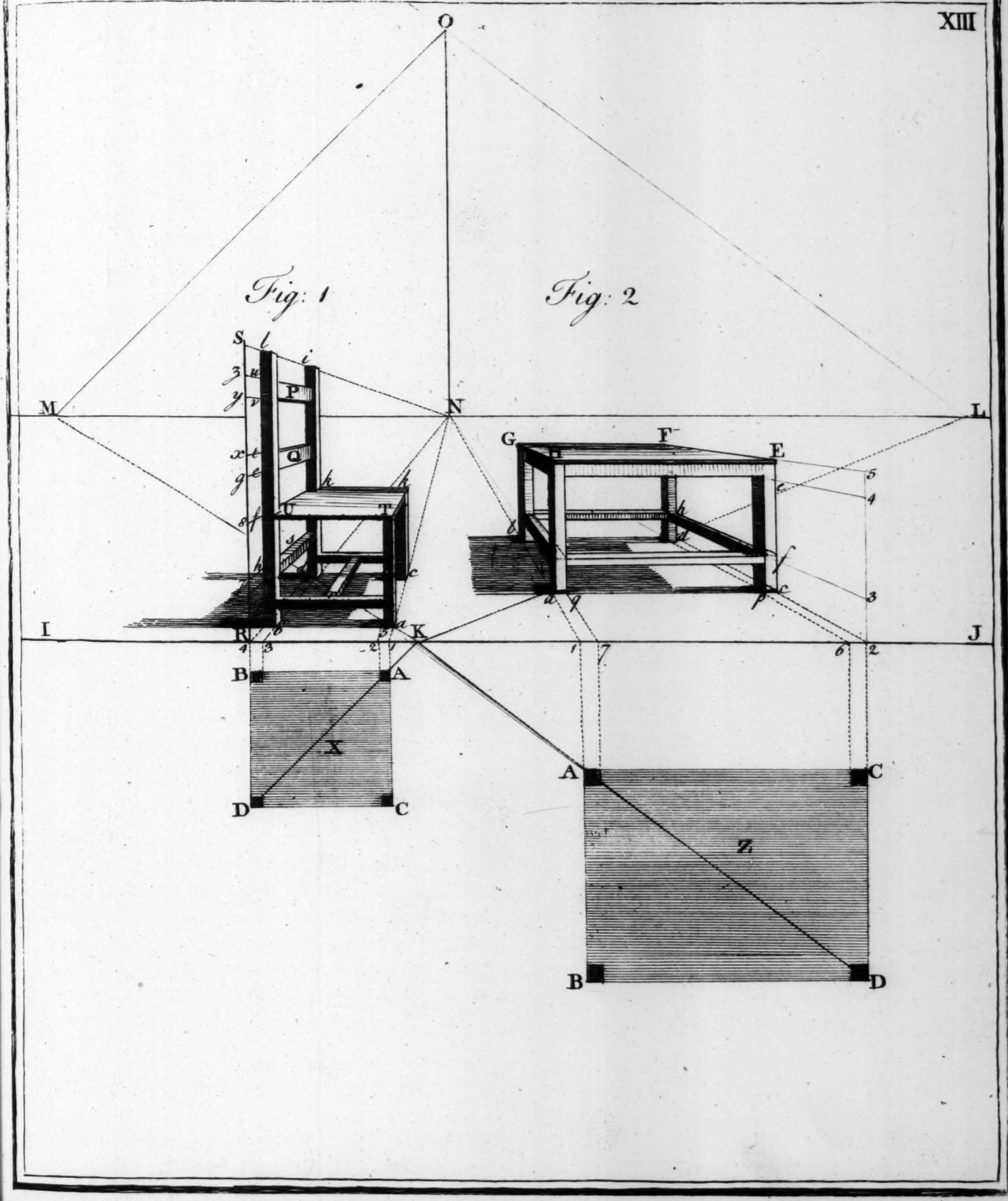
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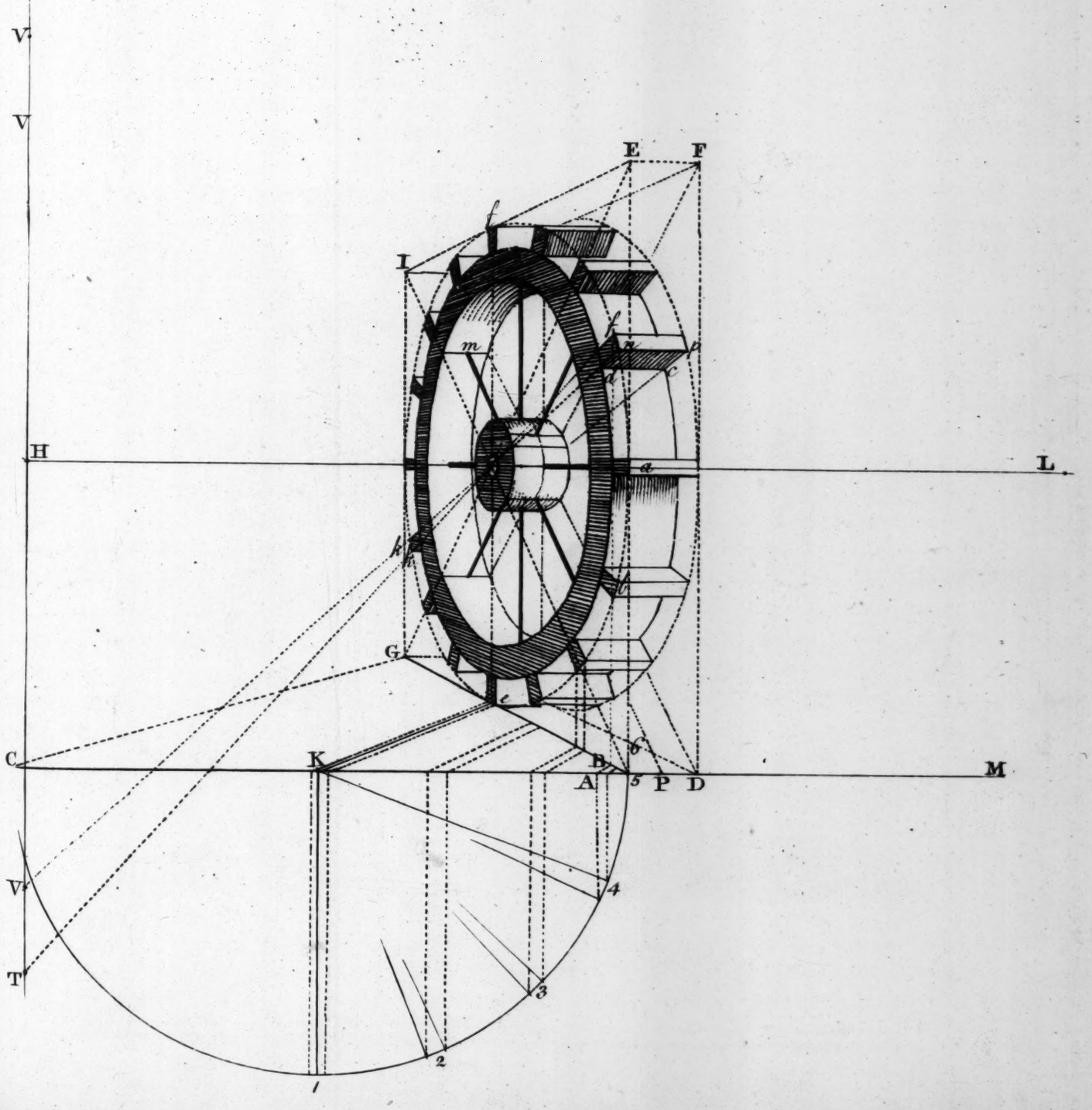
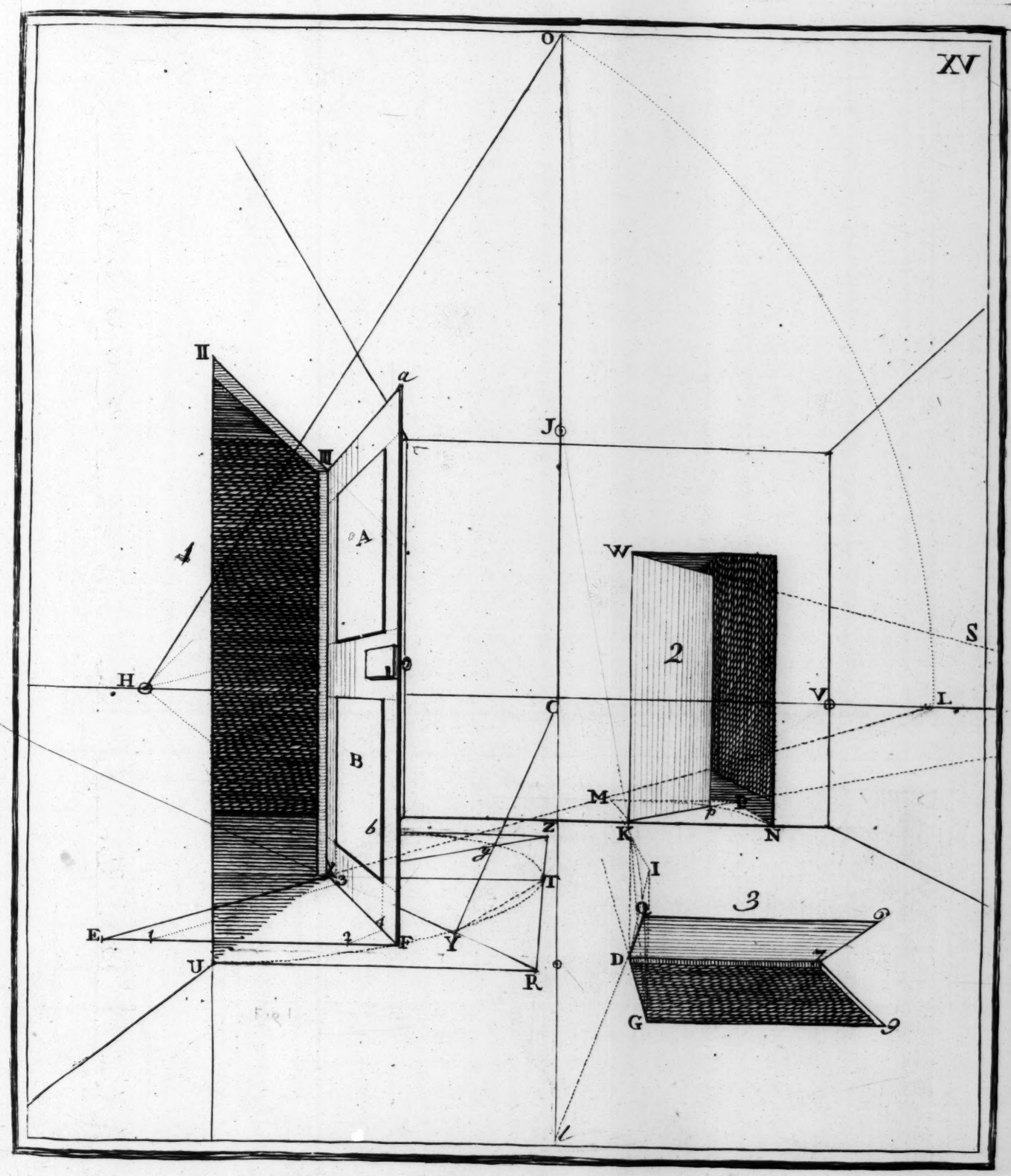
Pl. XIV.

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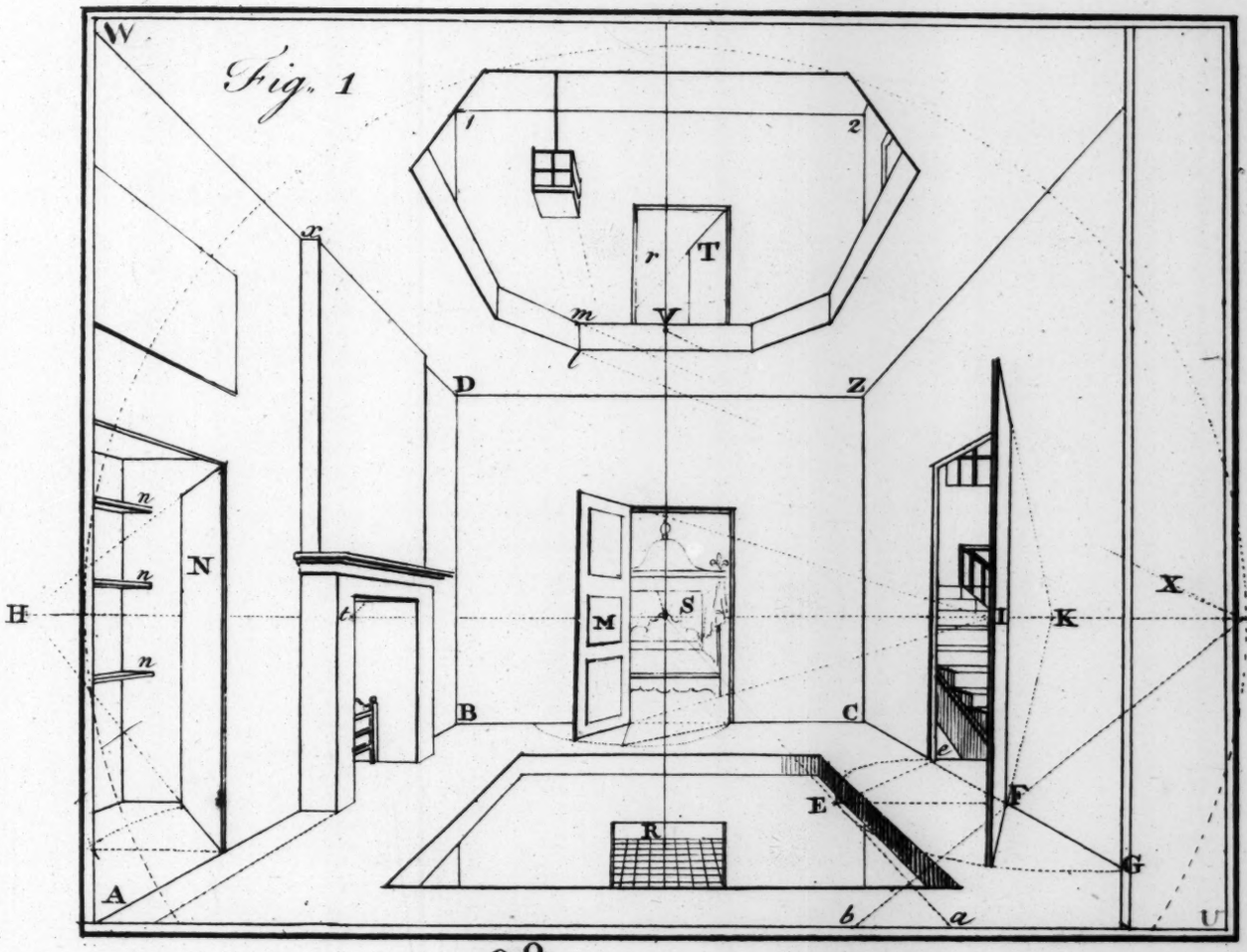
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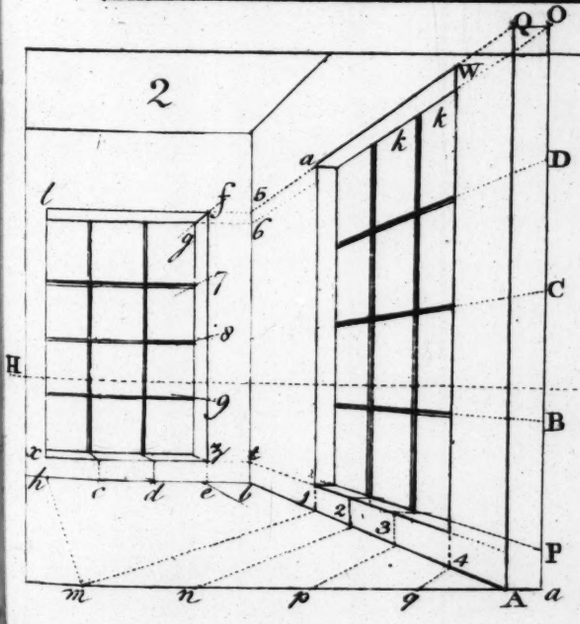
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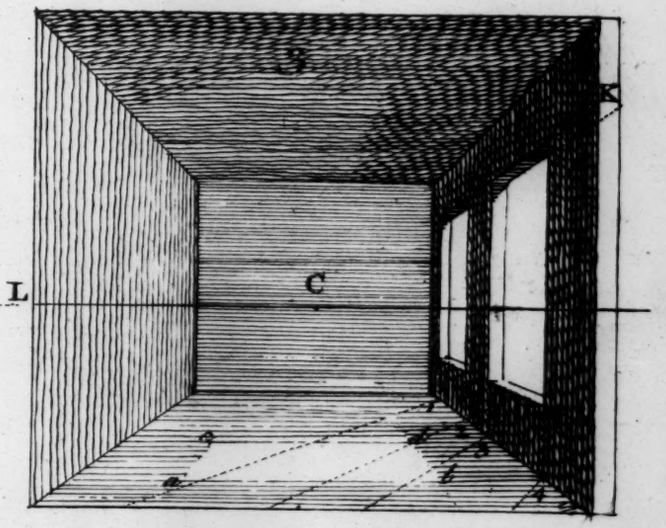
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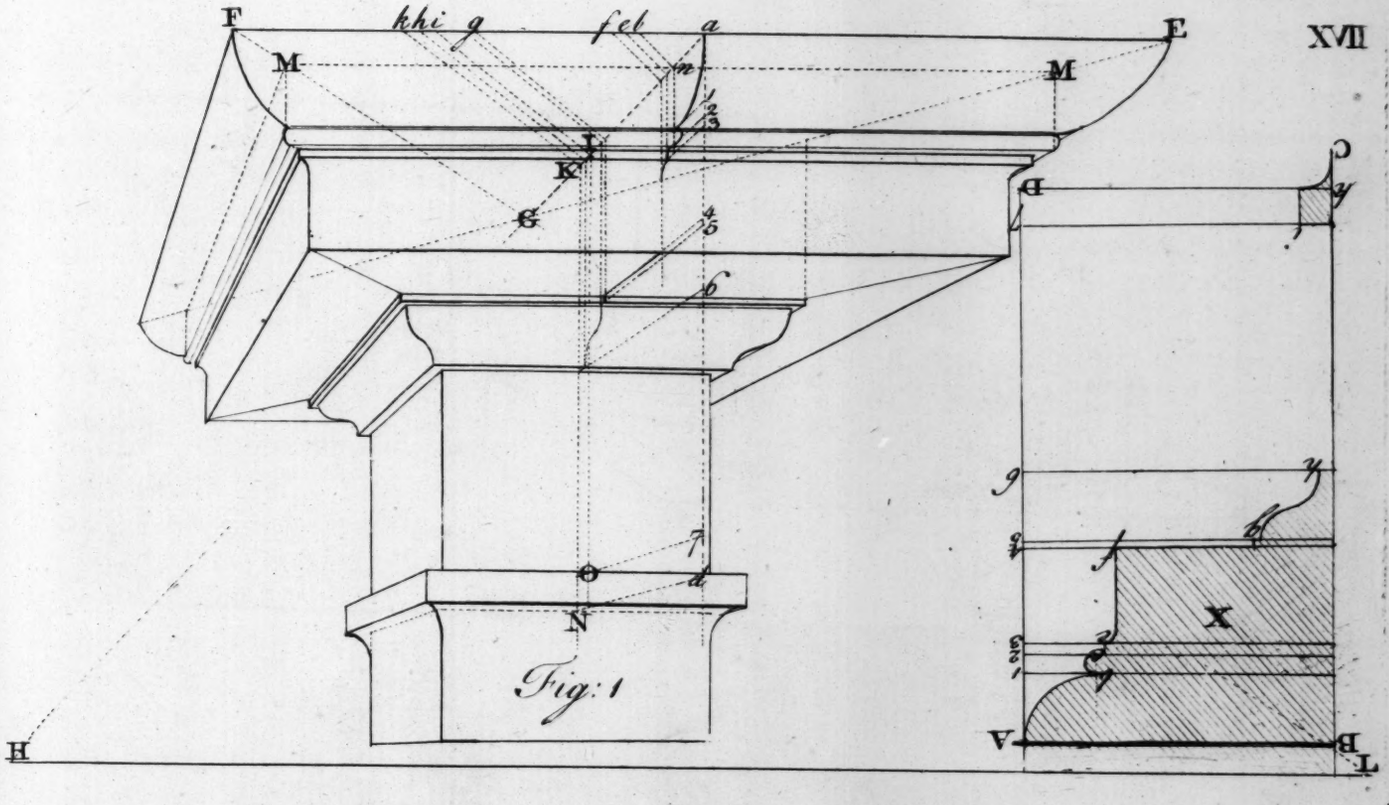

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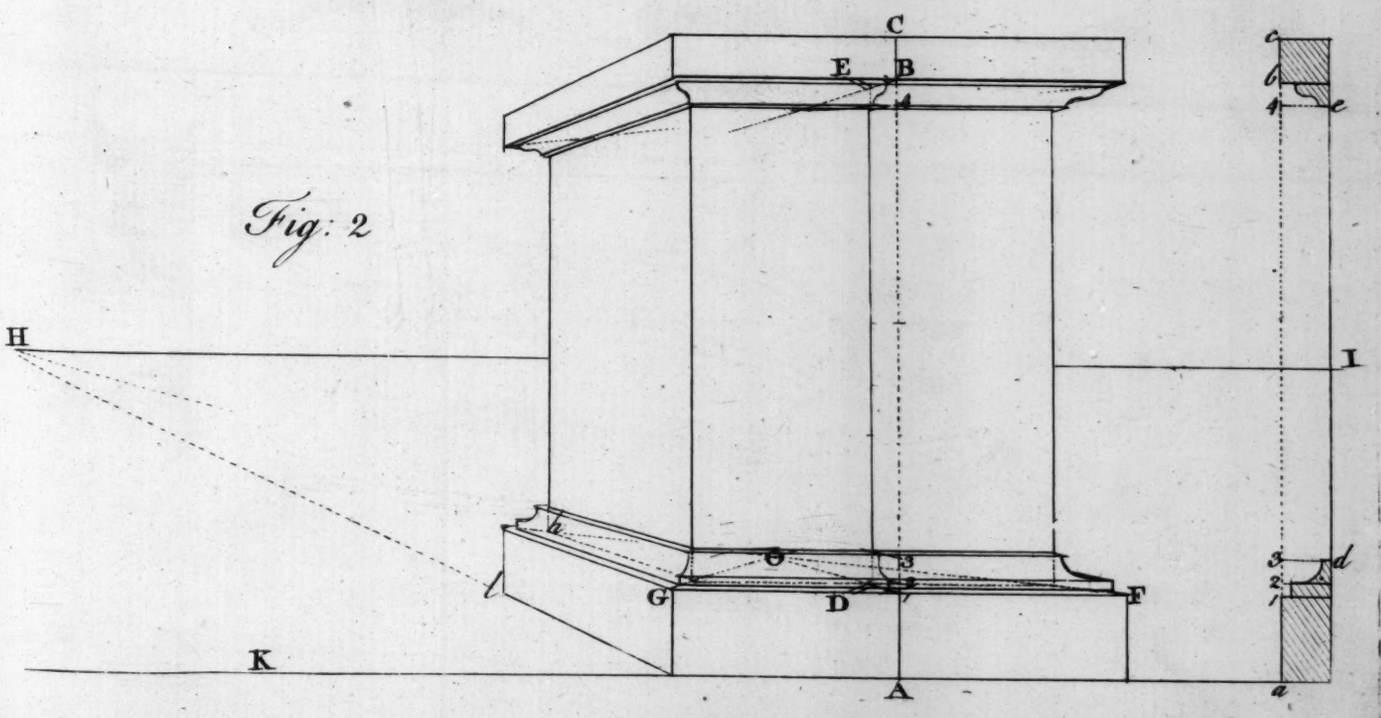

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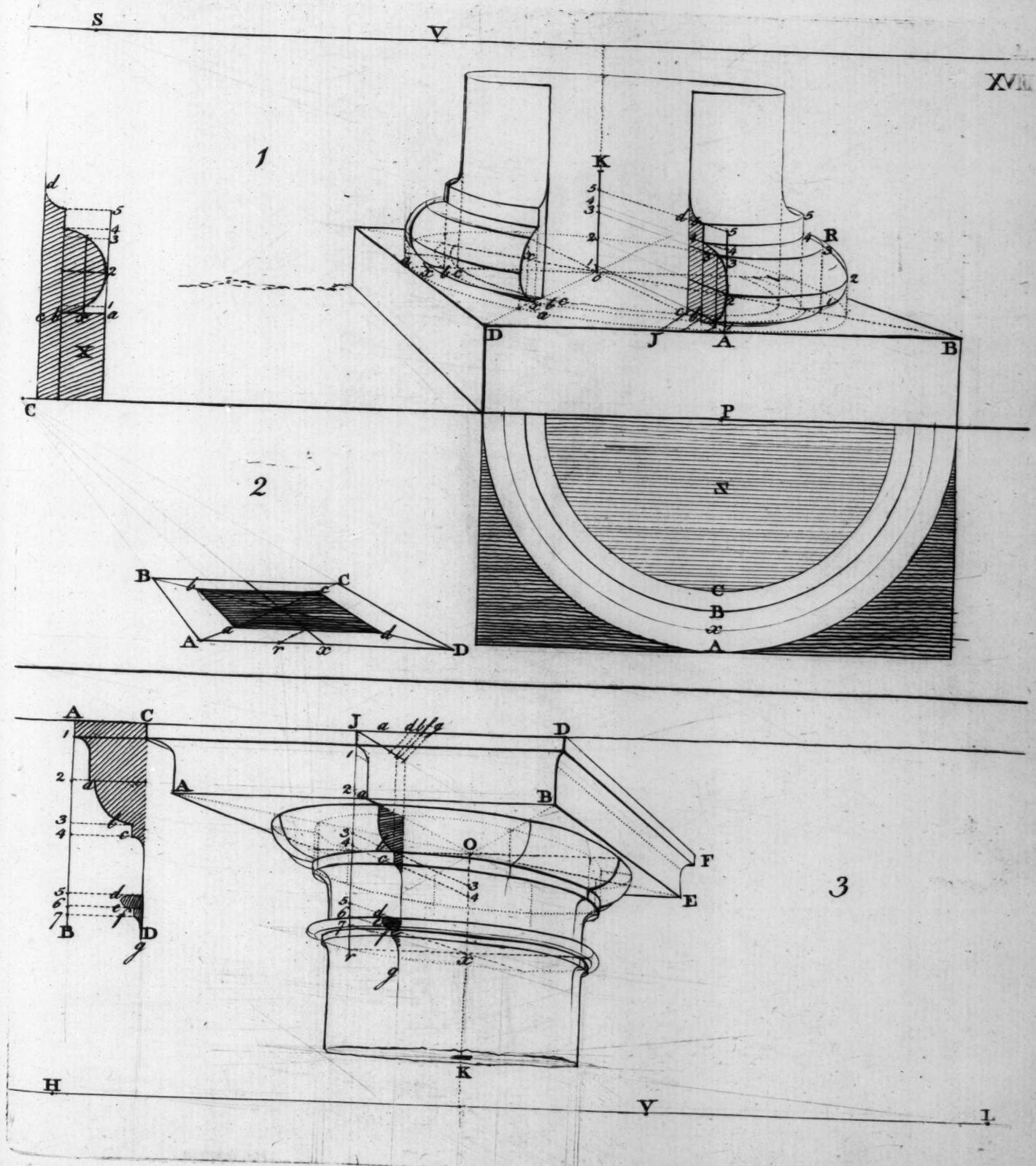
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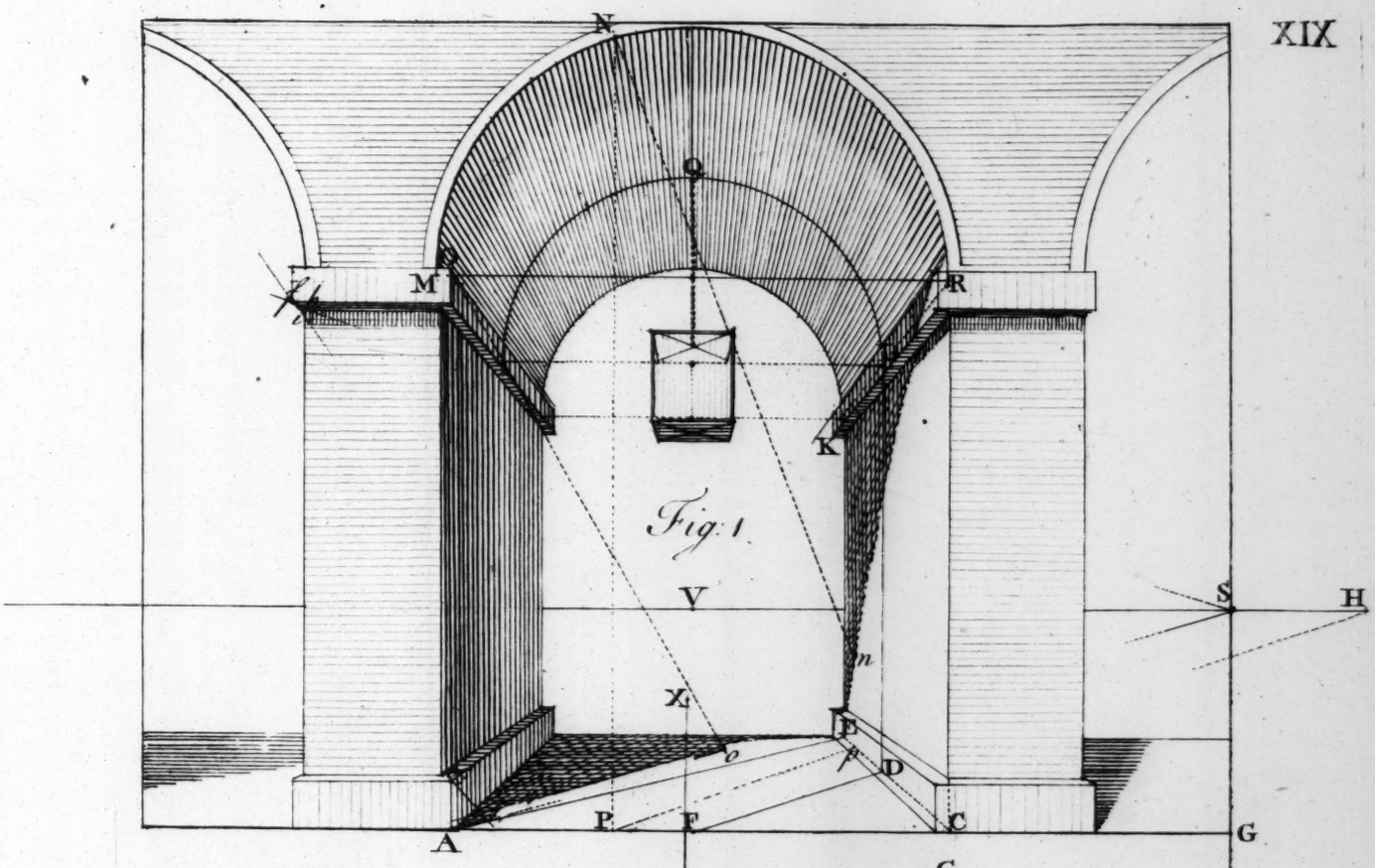

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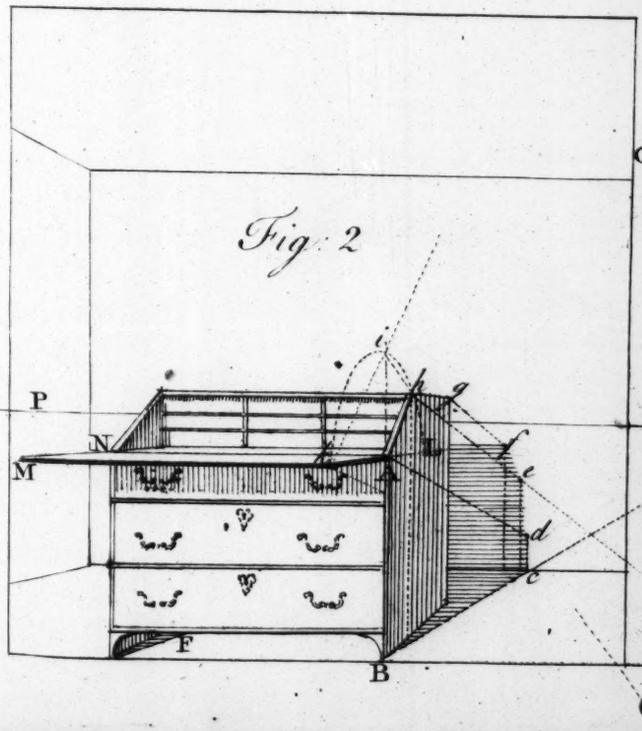

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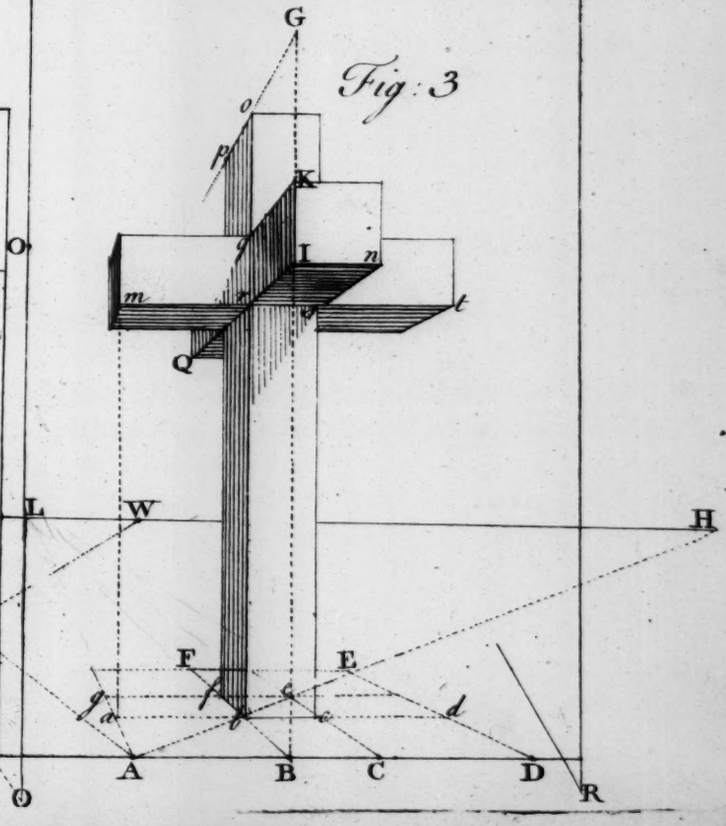

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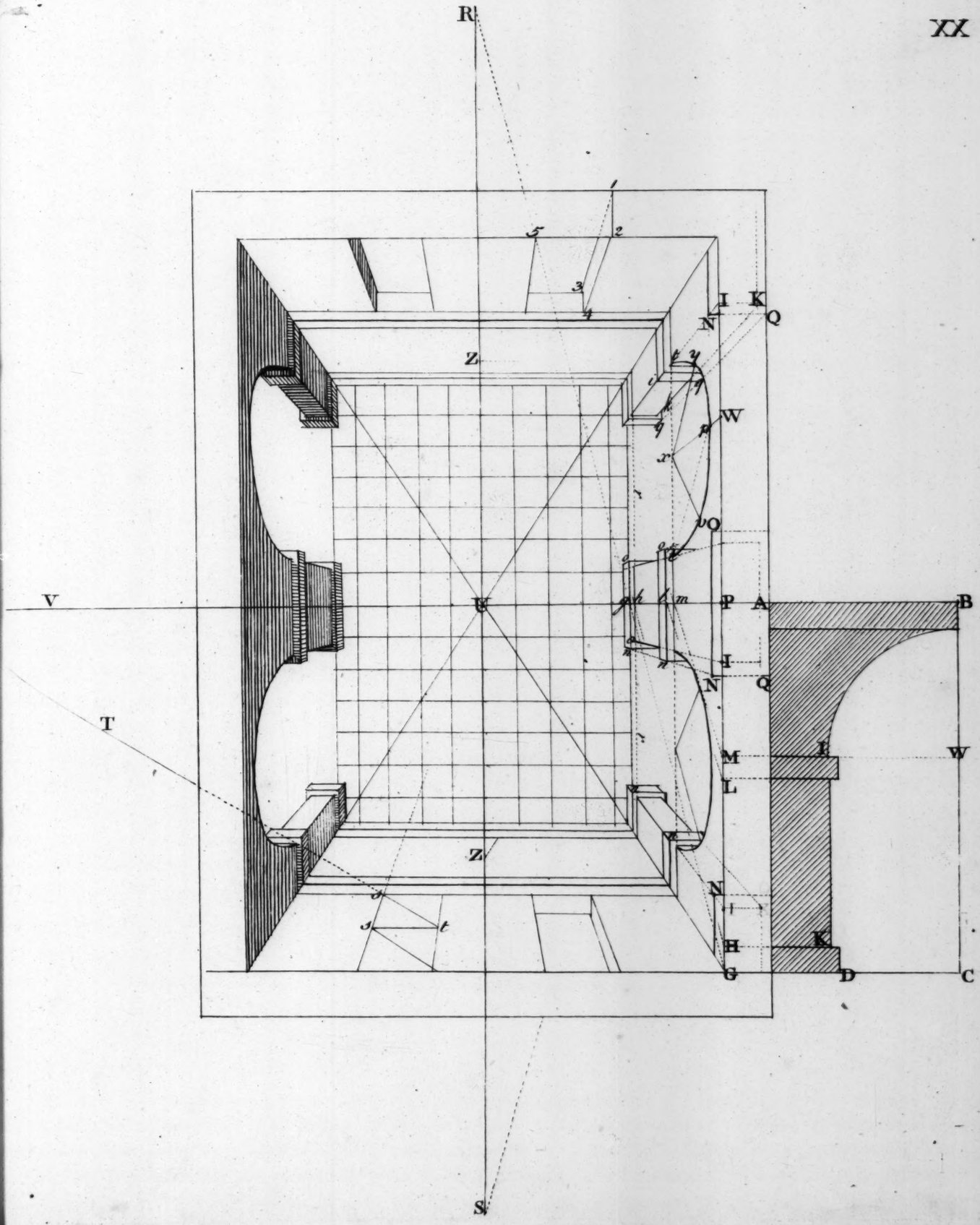
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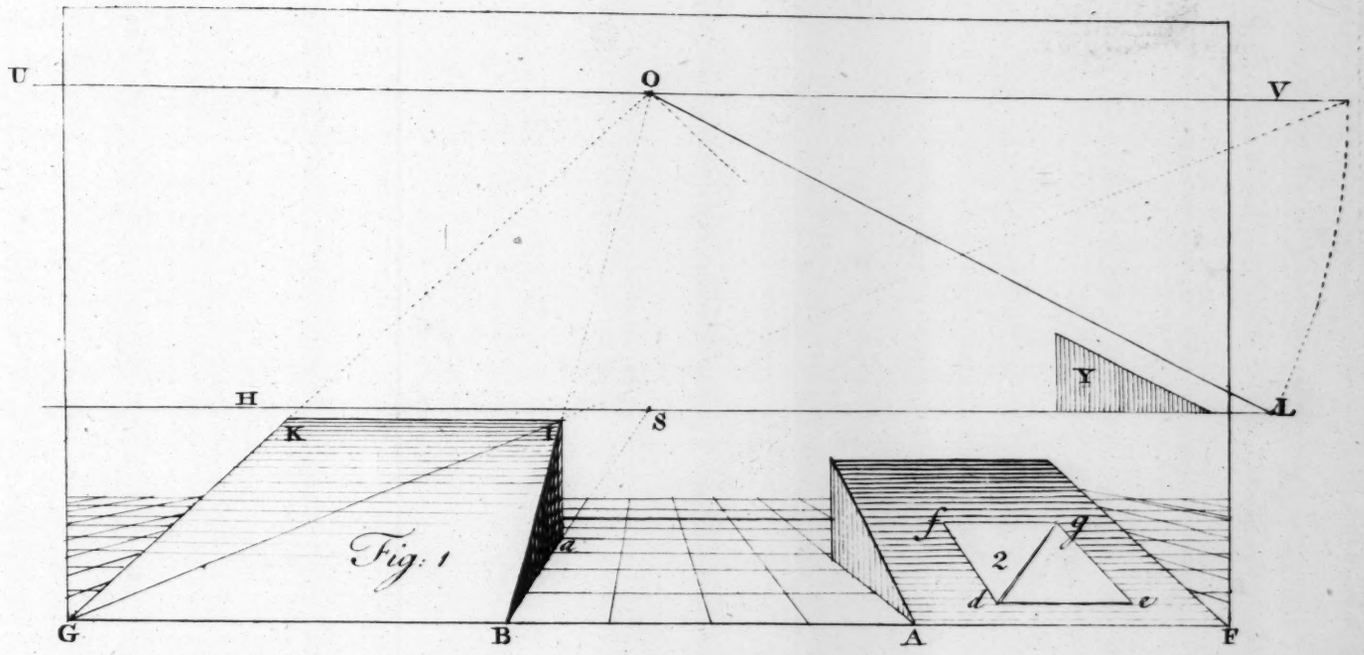

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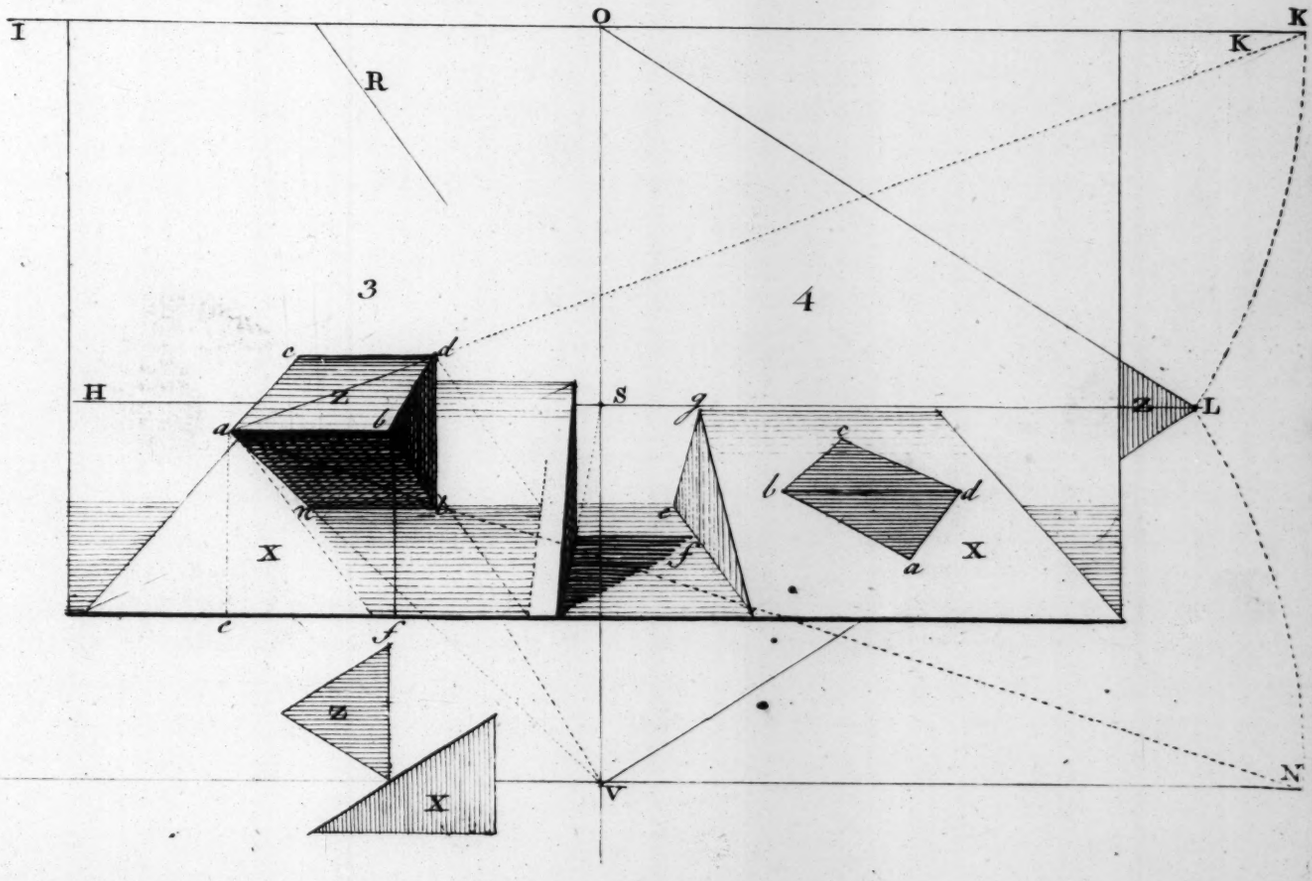

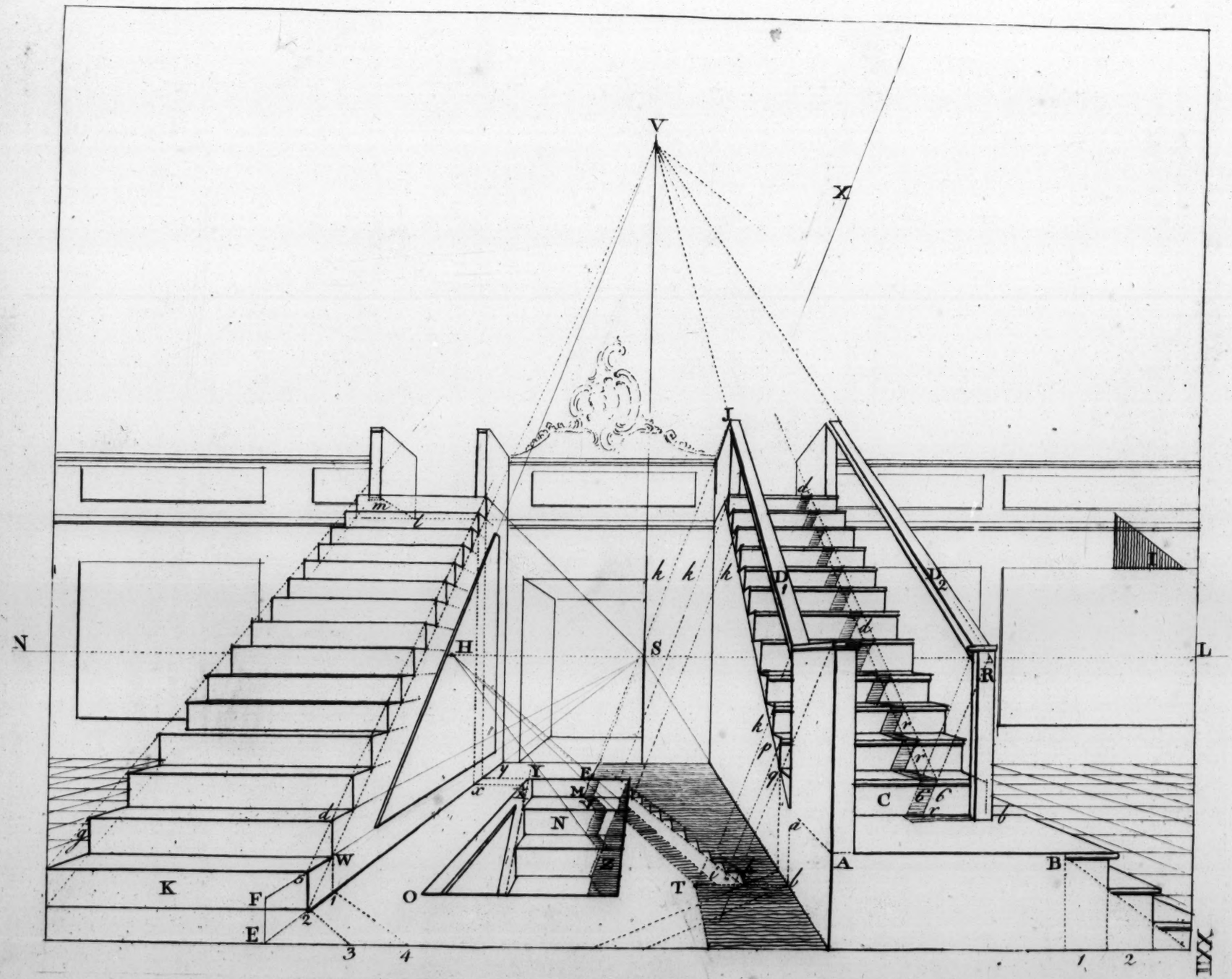

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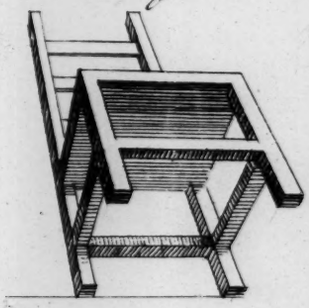

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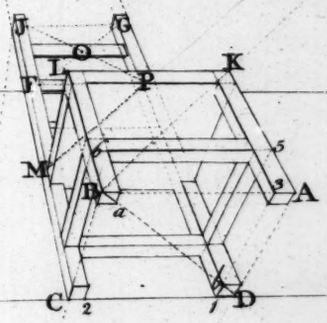

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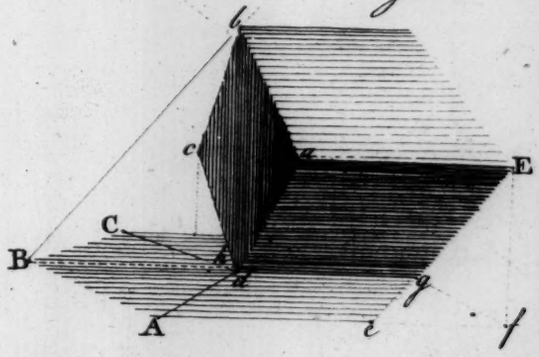

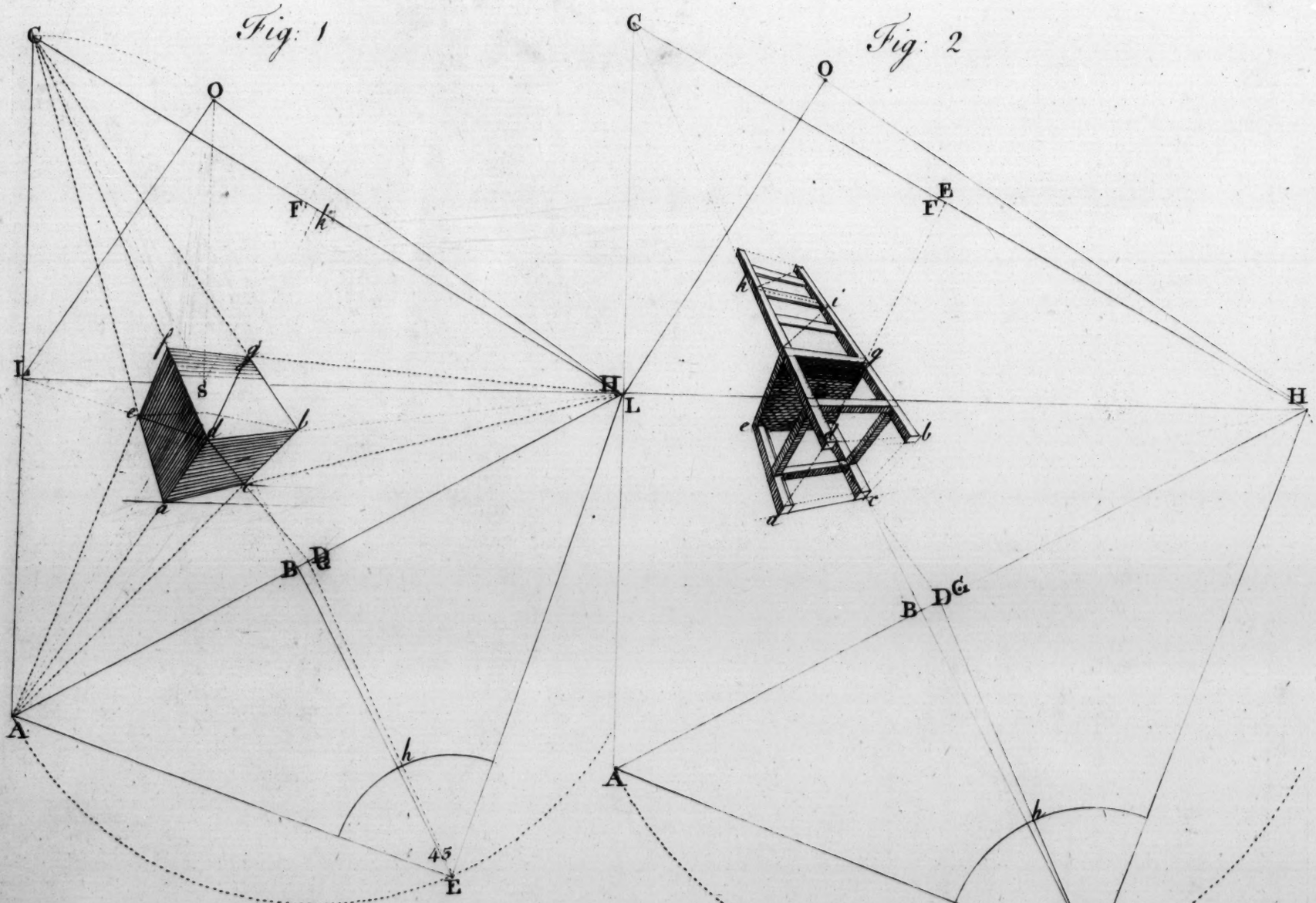
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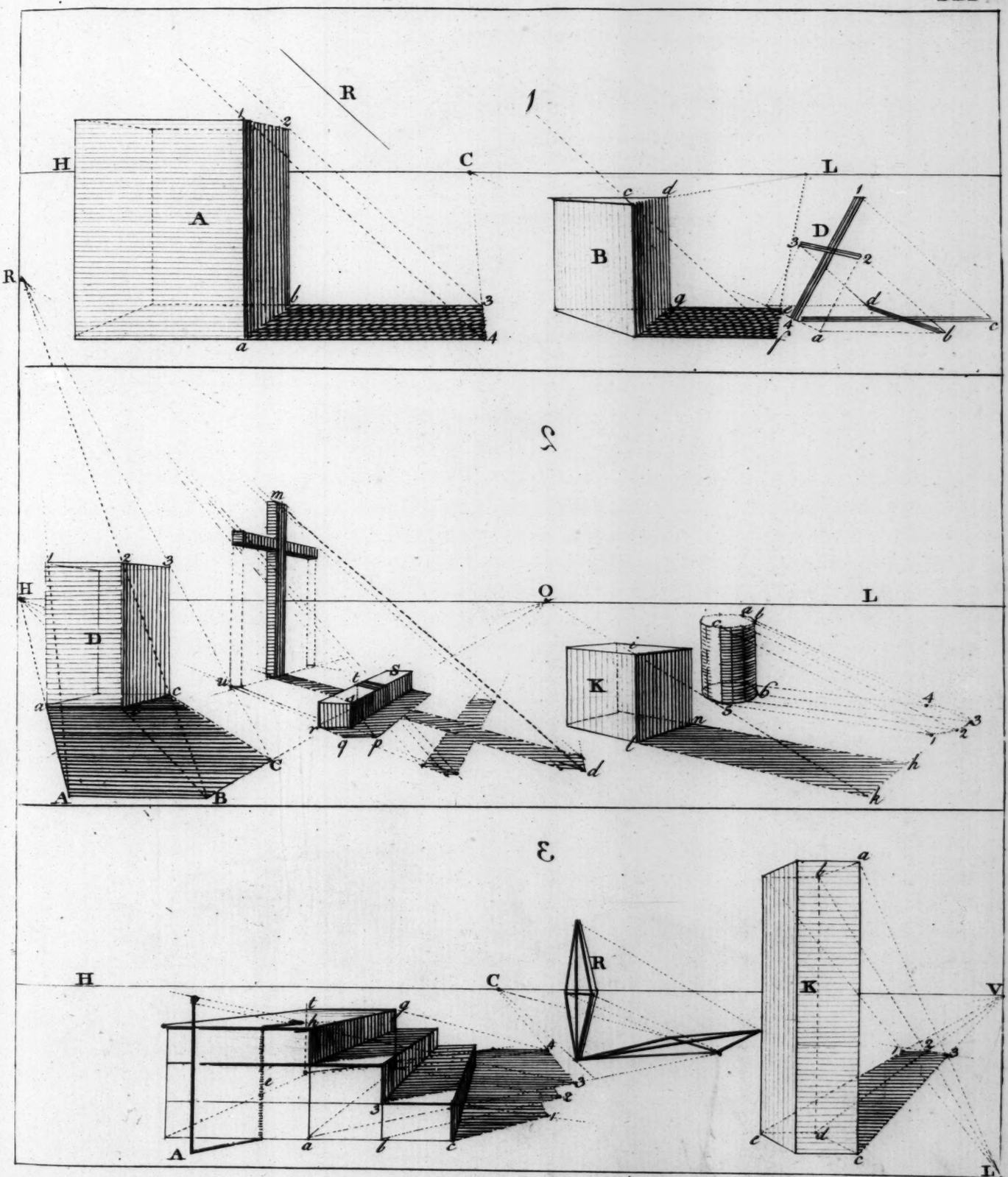
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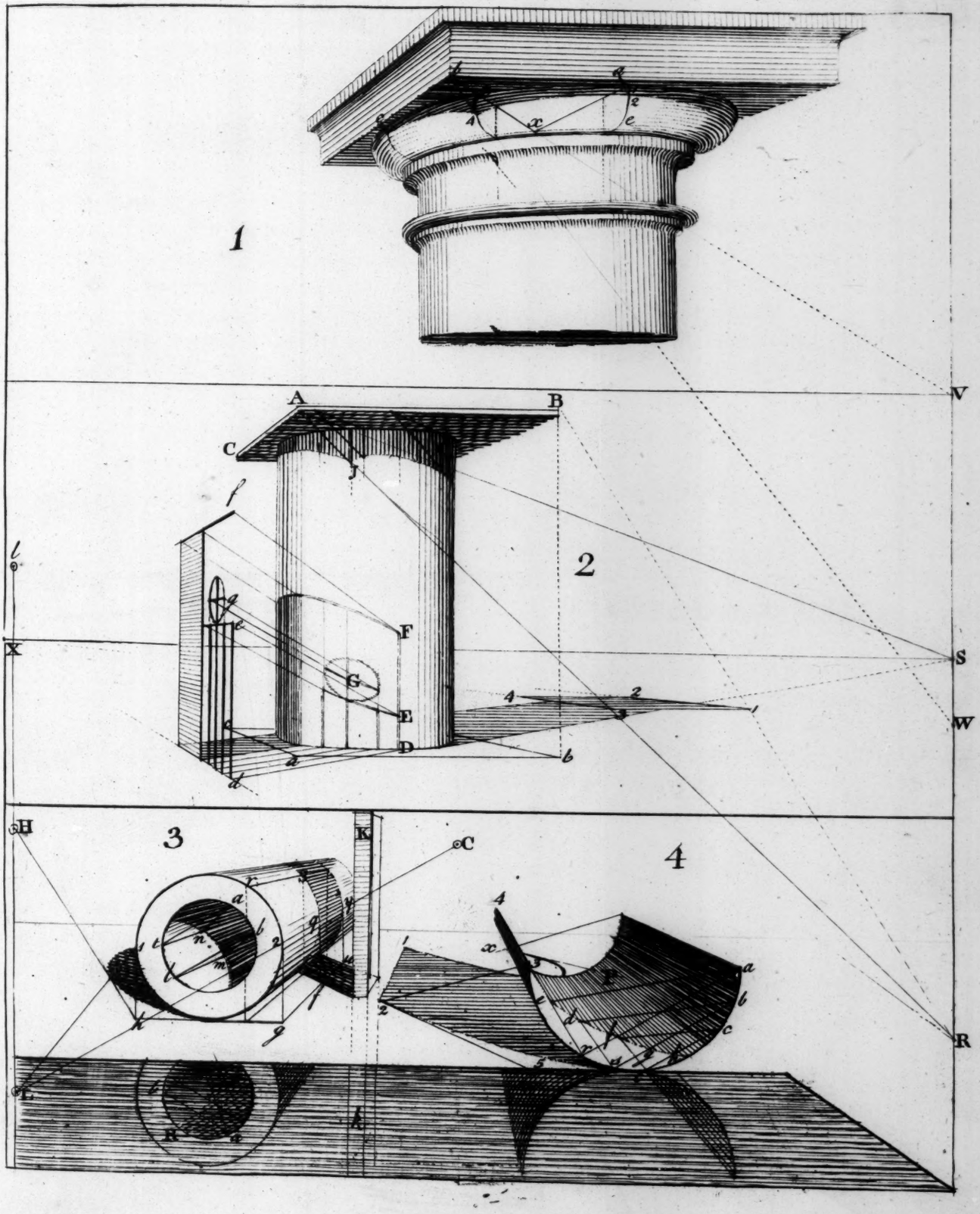
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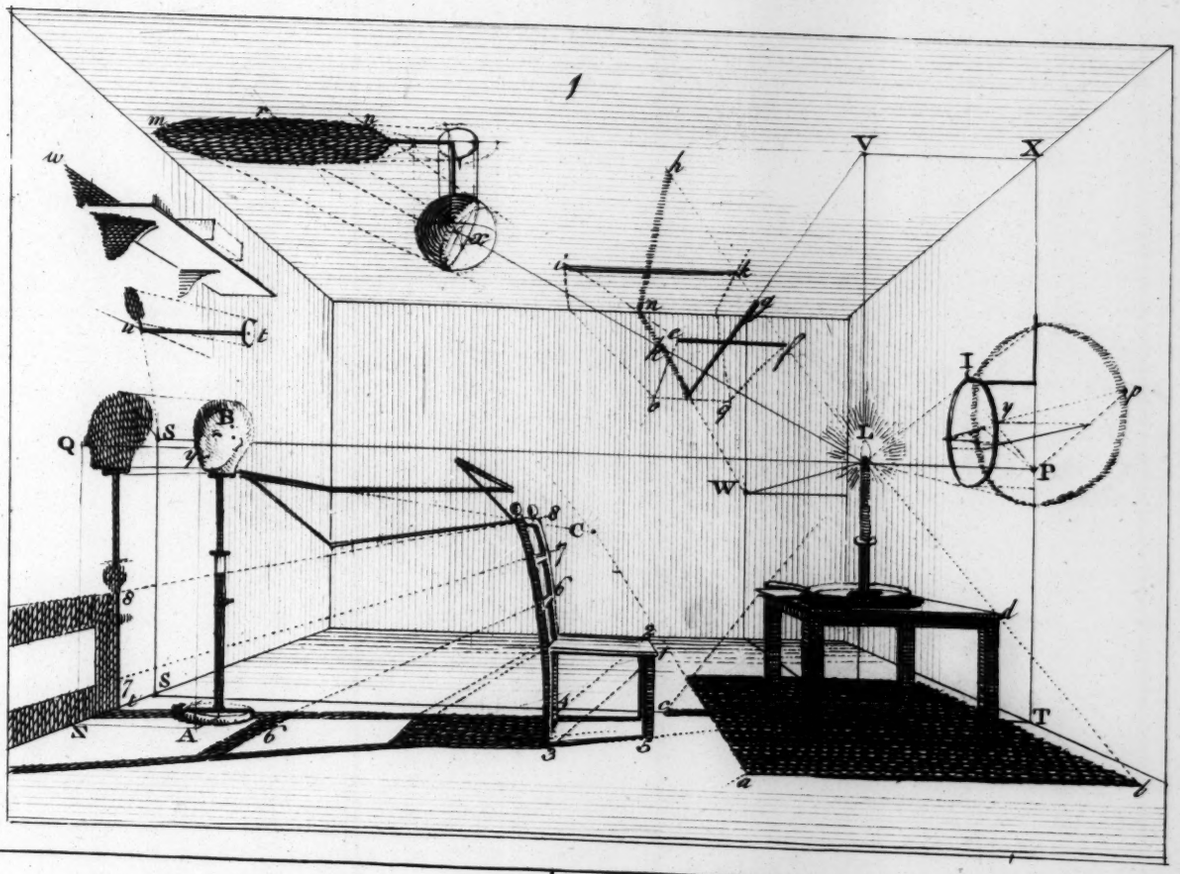

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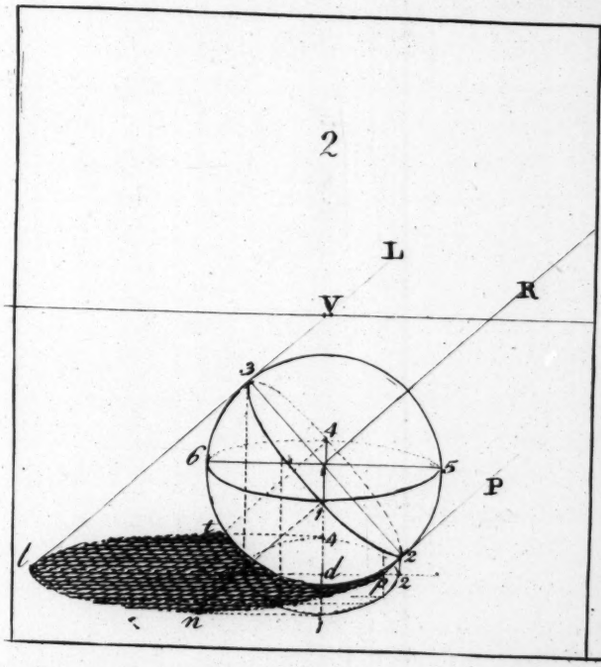

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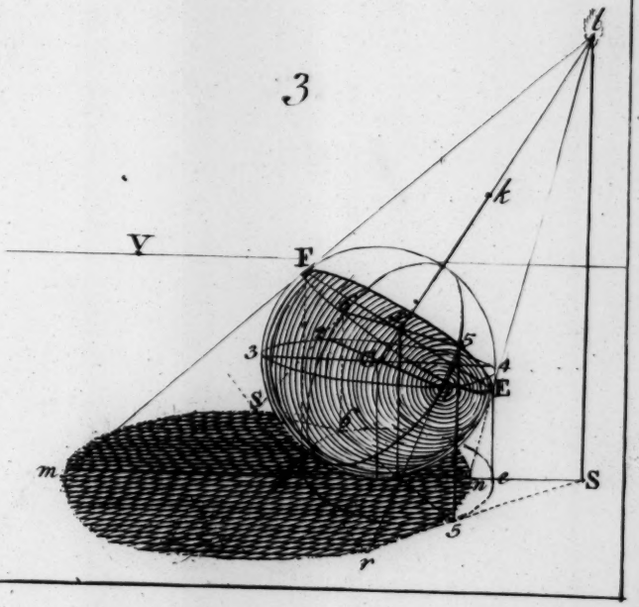

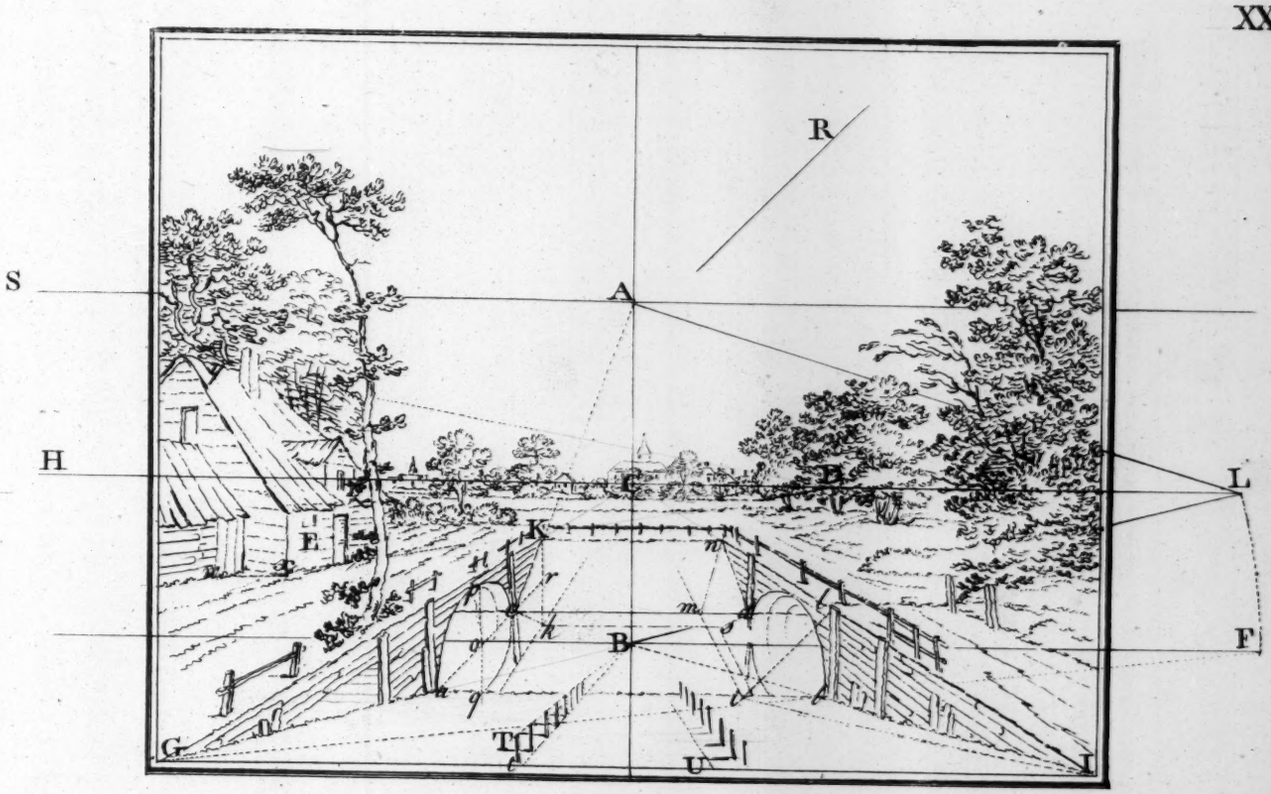
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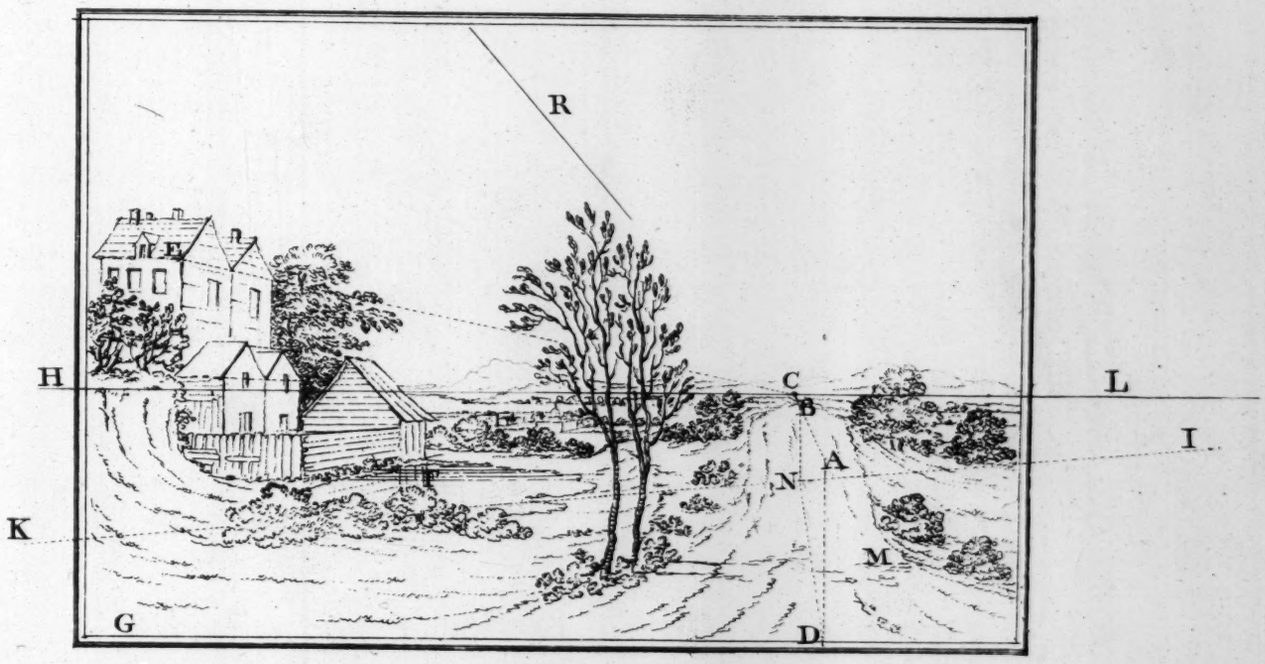
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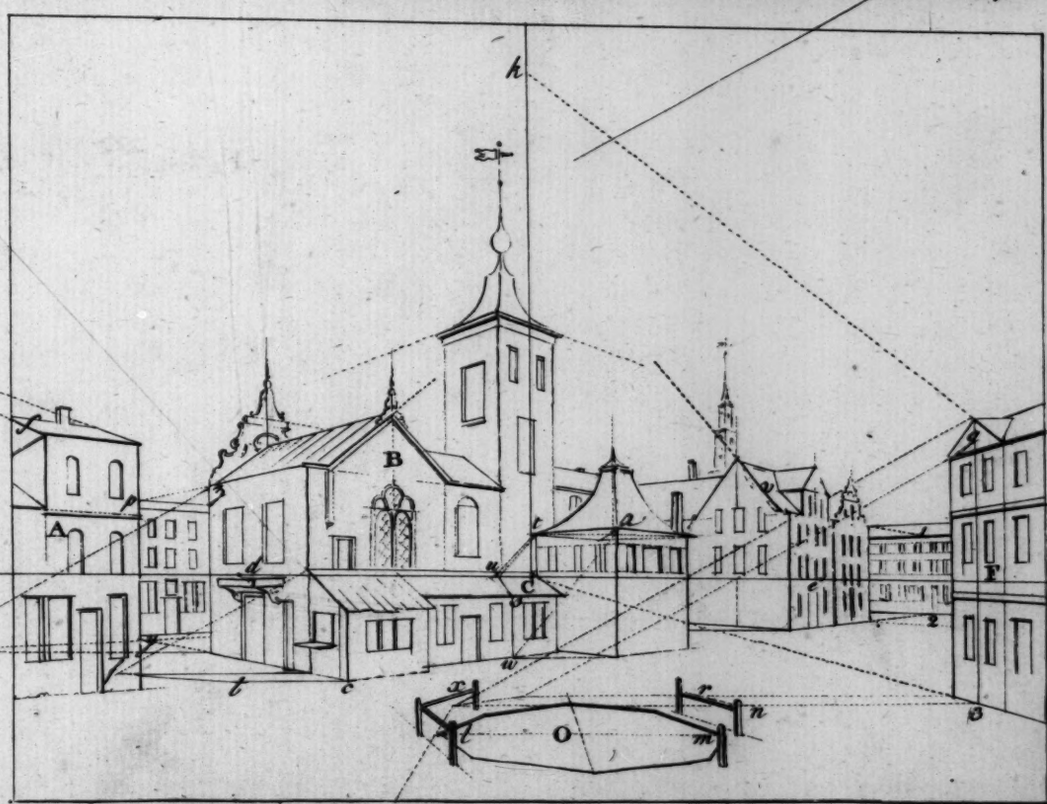
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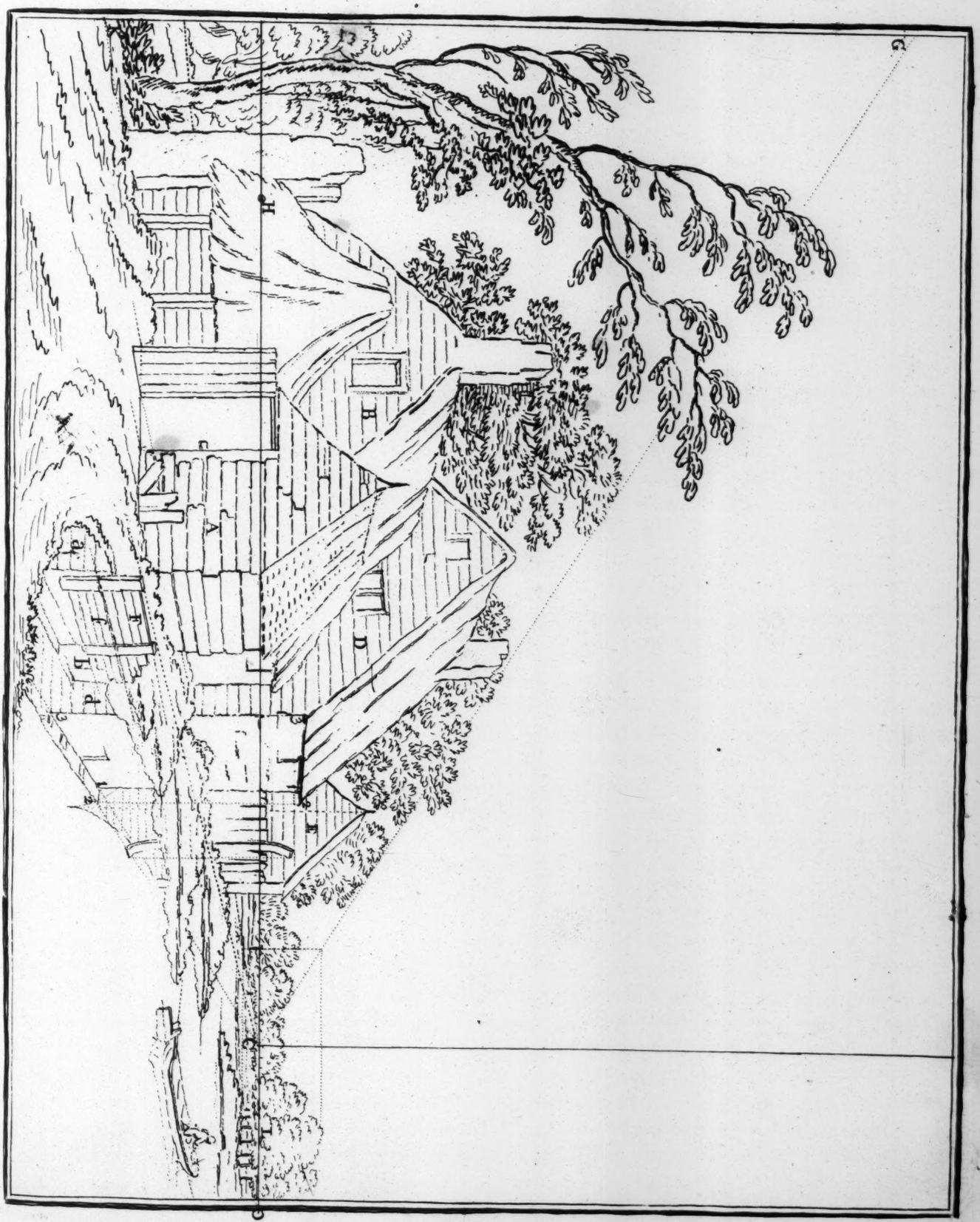
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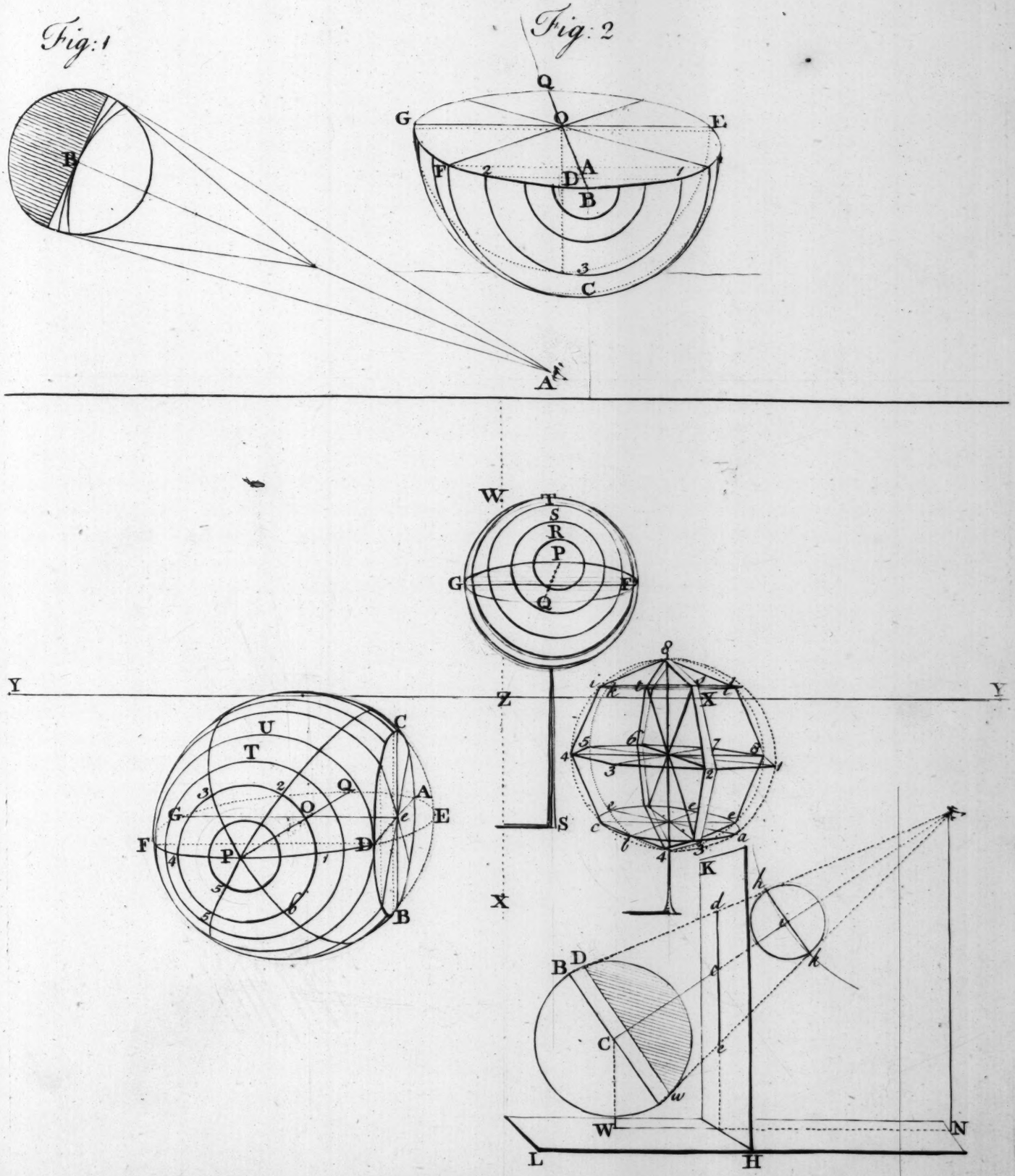
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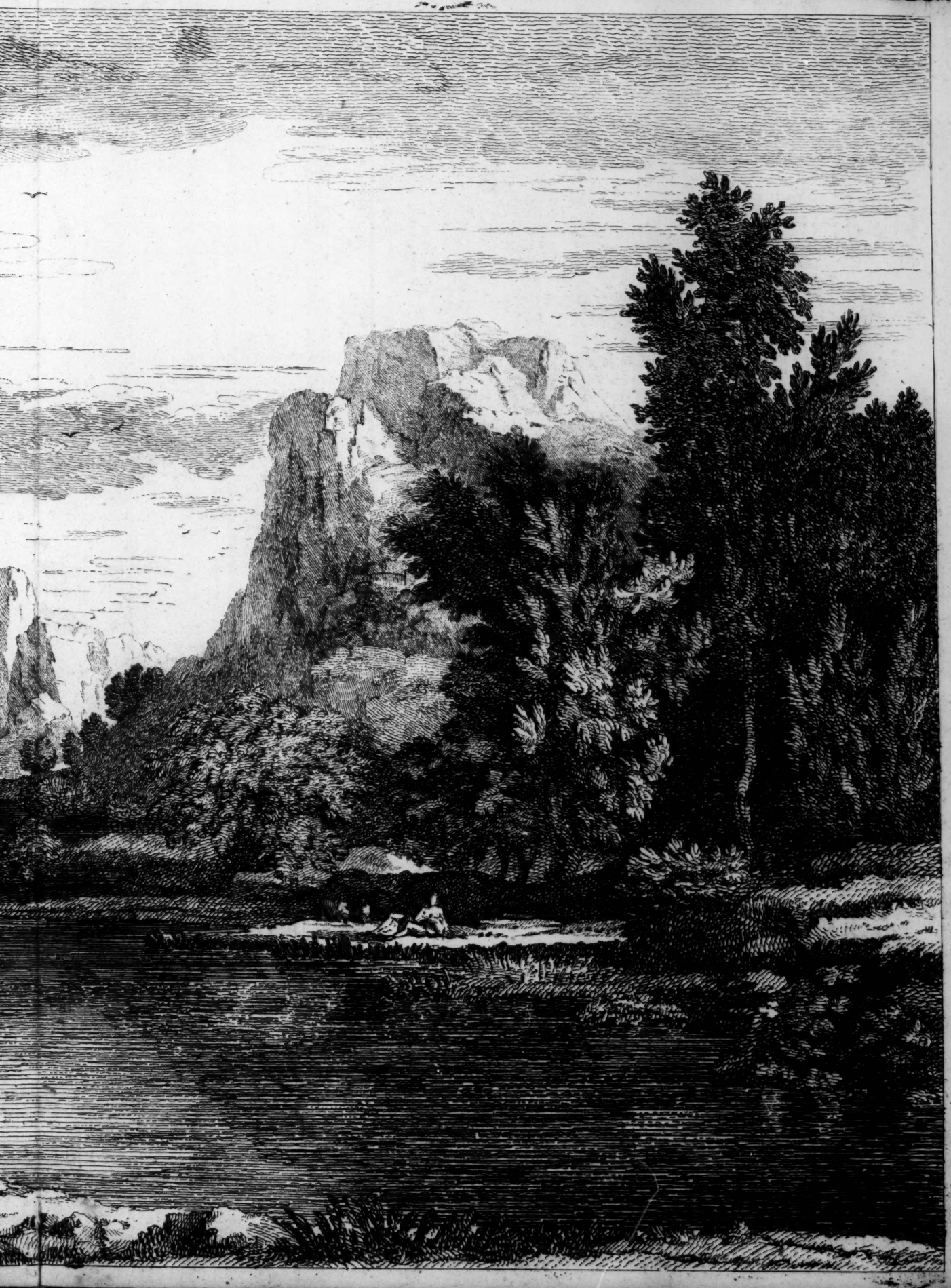

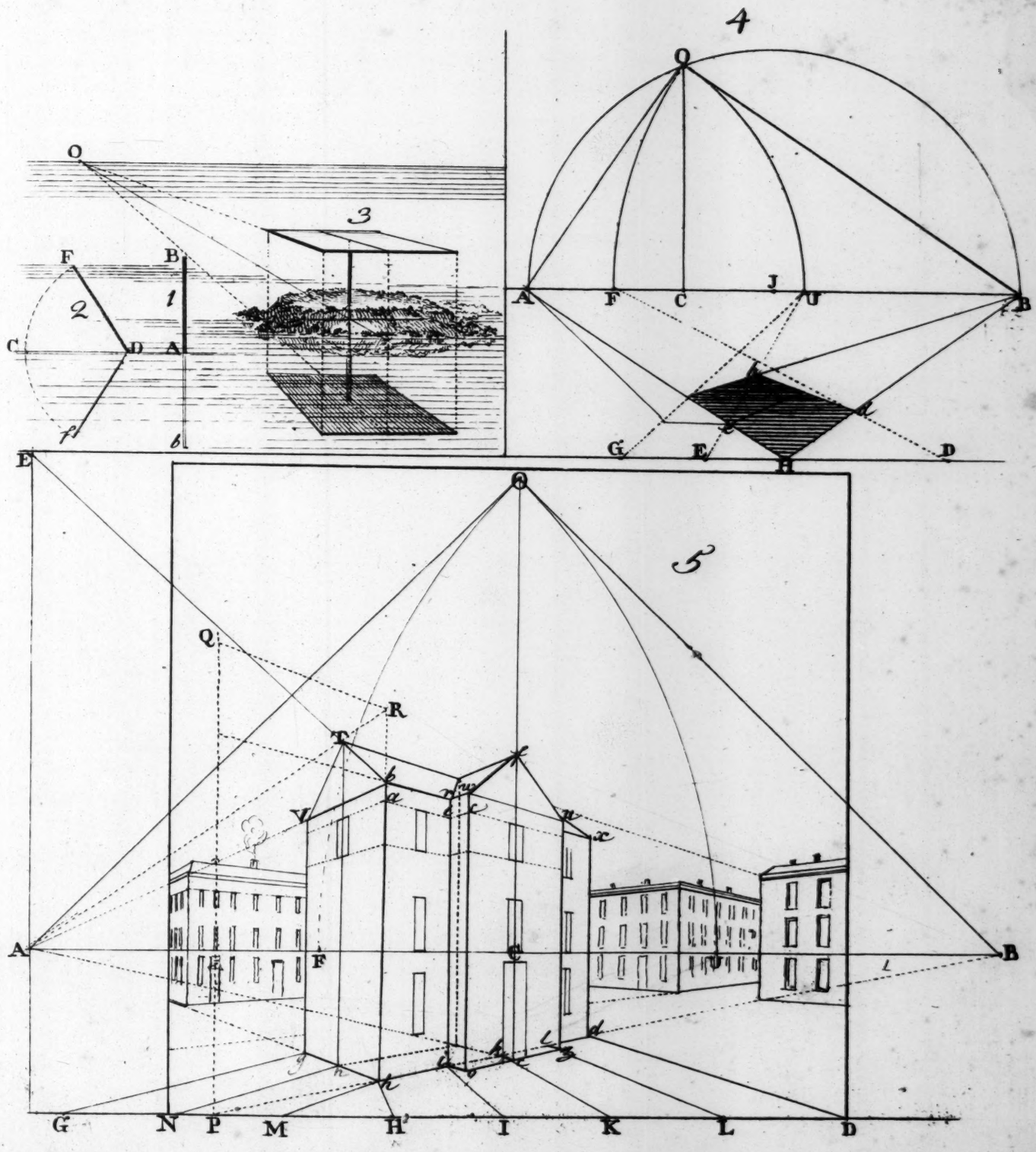
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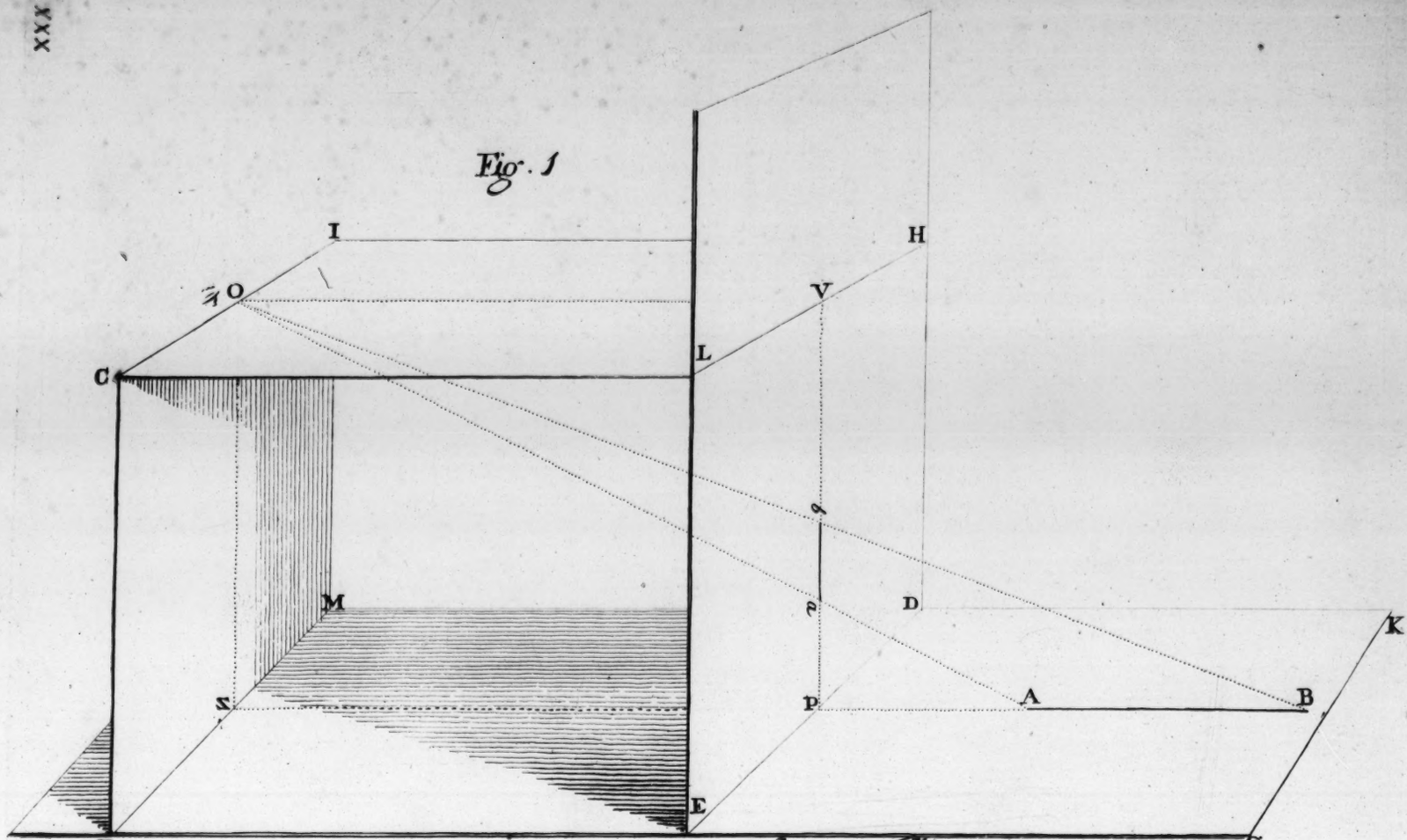

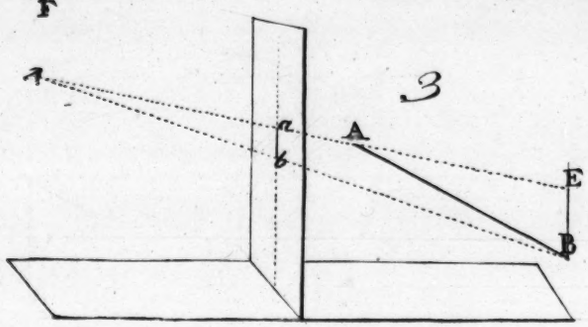

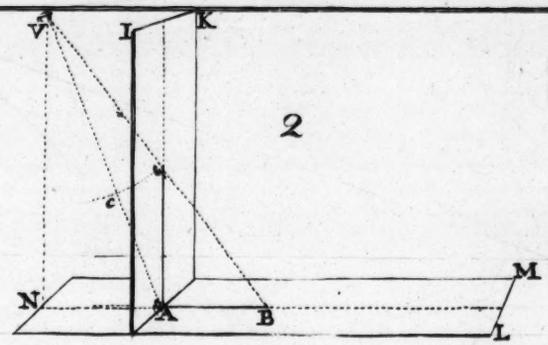

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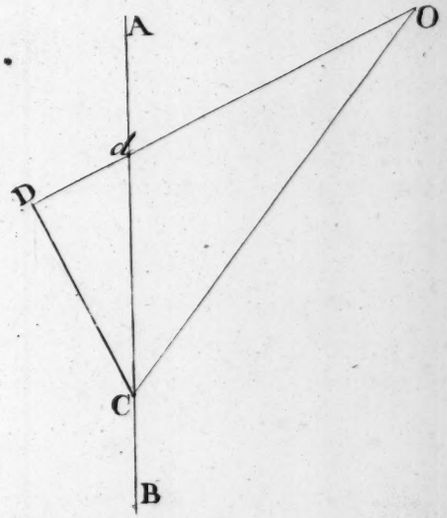

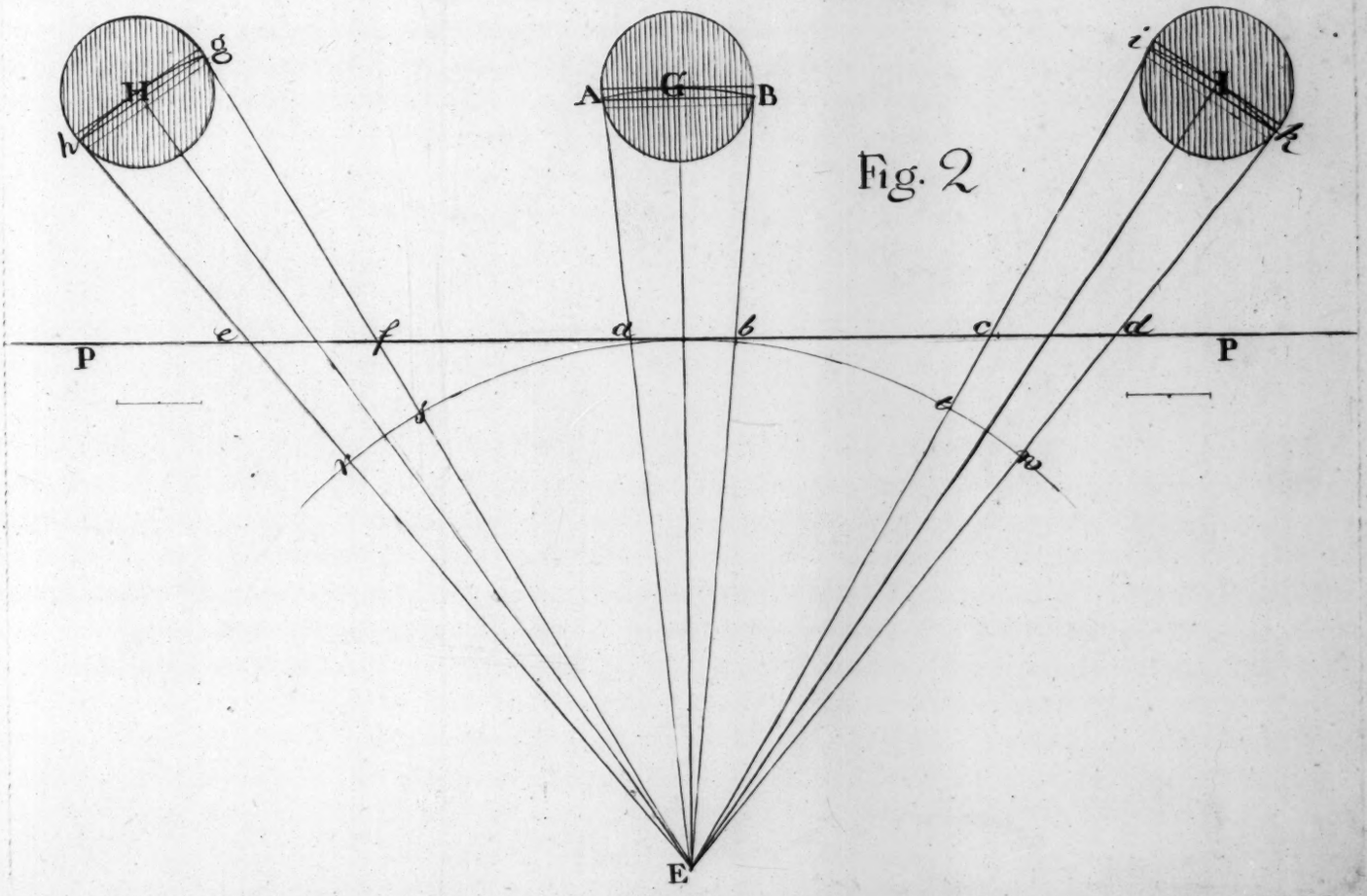

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